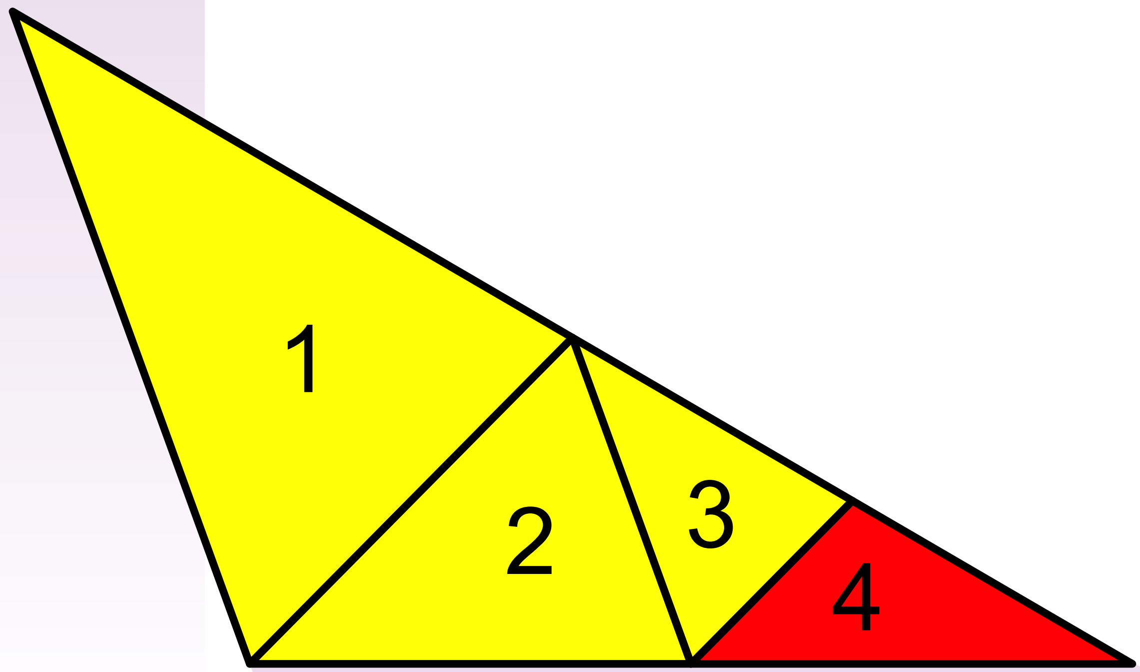
Geometrical Puzzles



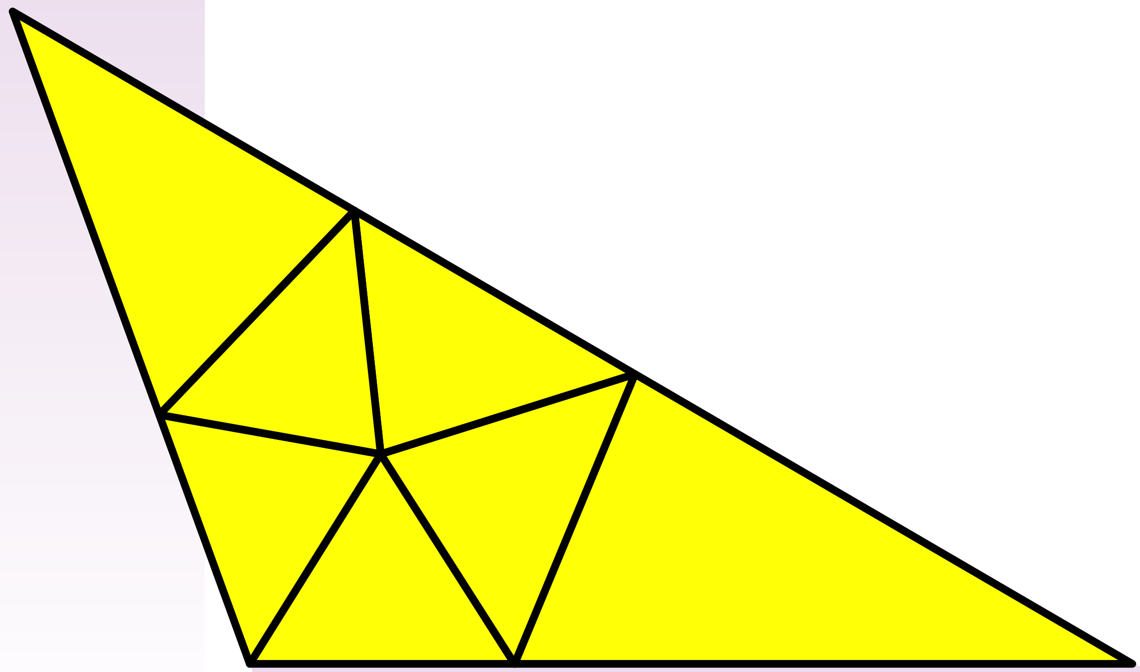
Acute Dissection

by Mel Stover

Is it possible to cut an obtuse triangle (a triangle with one obtuse angle) into smaller triangles, all of them acute? An acute triangle is a triangle with three acute angles. A right angle is neither acute nor obtuse. If such a dissection can be done, what is the smallest number of acute triangles into which any obtuse triangle can be dissected?

The illustration shows how an obtuse triangle can be divided into almost all acute triangles except one - the red one. Thus what approach should be used when it is required to cut an obtuse triangle into acute triangles *only*?

Geometrical Puzzles



Acute Dissection (solution)

The minimal number of acute triangles is 7. The pattern for the seven triangles is shown in the illustration. An elegant proof for the seven acute triangles has been provided by Wallace Manheimer in *American Mathematical Monthly*, November 1960. The logic behind the proof is as follows.

The obtuse angle must be divided by a line. This line cannot go all the way to the other side, for then it would form another obtuse triangle (or two triangles with right angles), which in turn would have to be dissected, consequently the pattern for the large triangle would not be minimal. The line dividing the obtuse angle must, therefore, terminate at a point inside the triangle. At this vertex, at least five lines must meet, otherwise the angles at this vertex would not all be acute. This creates the inner pentagon of five triangles, making a total of seven triangles as shown in the illustration.