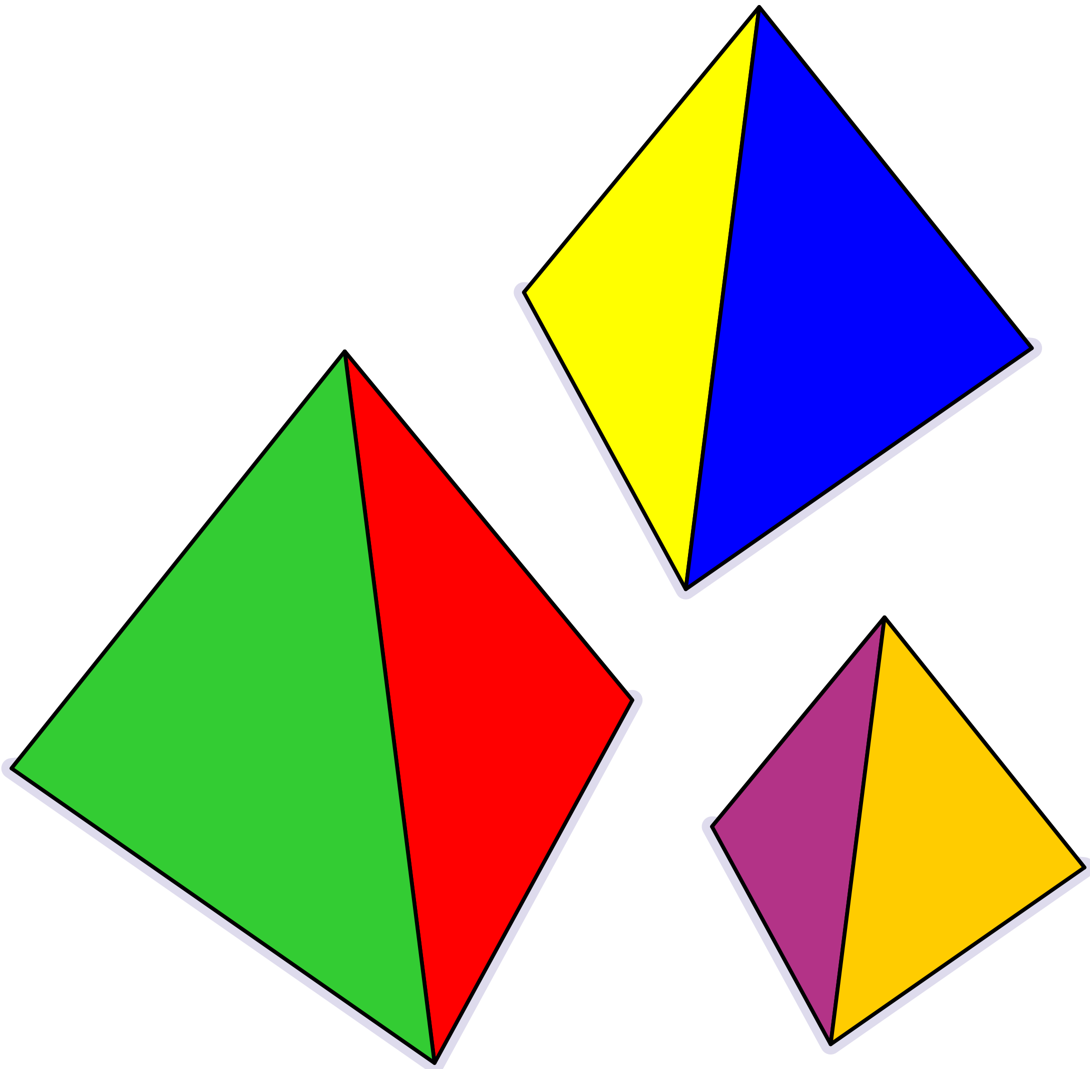

There are seven primary colors of the solar spectrum - violet, indigo, blue, green, yellow, orange, and red (or "Vibgyor").

This puzzle concerns the painting of the four sides of a tetrahedron, or triangular pyramid. Each time no more than four colors from the solar spectrum can be used to paint a pyramid.

The question is in how many unique ways may the triangular pyramid be colored, using in every case one, two, three, or four colors of the solar spectrum? A side can only receive a single color, and no side can be left uncolored. The crucial point of the challenge is careful selection of the painting scheme in order to avoid the repetitions of the pyramids. In other words if a colored pyramid cannot be placed so that it exactly resembles in its colors and their relative order another pyramid, then they both are different. Otherwise they are the same. Remember that one way would be to color all four sides red, another to color two sides green, and the remaining sides yellow and blue; and so on.

Painting a Pyramid
Solution

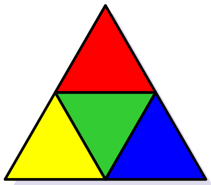
Fig. 1

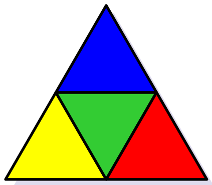
Fig. 2

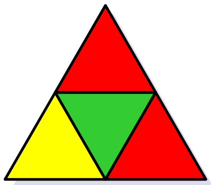
Fig. 3

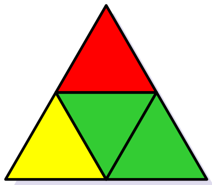
Fig. 4

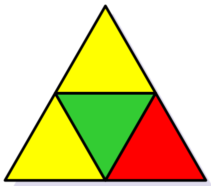
Fig. 5

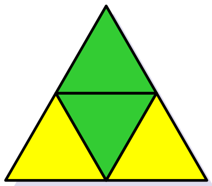
Fig. 6

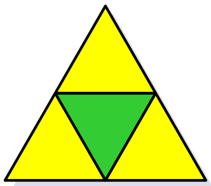
Fig. 7

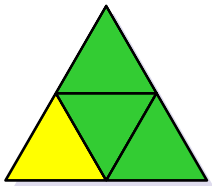
Fig. 8

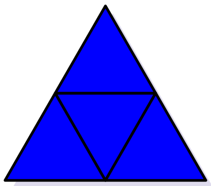
Fig. 9

We have a set of seven colors - Vibgyor. Now the question is how many combinations (subsets) of 4, 3, 2, and 1 colors can be selected from the 7-color set? Suppose, at this stage the order of the numbers in the subset doesn't matter. It means the subset of four colors "blue-green-red-yellow" is considered to be the same as the subset "green-blue-yellow-red."

There is a formula how to figure out the amount of unordered combinations of X with Y numbers. It is equal to Y!/(X! (Y-X)!). X! is factorial of X, Y! is factorial of Y and (Y-X)! is factorial of (Y-X). Any factorial is a product of consecutive numbers starting from 1 till the final number inclusive. For example, 4! equal to 1x2x3x4 or 24.

Thus, the amount of unordered combinations of X with Y is:
1) 4 numbers: 7!/(4! x (7-4)!) = 5040/(24 x 6) = 35;
2) 3 numbers: 7!/(3! x (7-3)!) = 5040/(6 x 4) = 35;
3) 2 numbers: 7!/(2! x (7-2)!) = 5040/(2 x 120) = 21;
1) 1 number: 7!/(1! x (7-1)!) = 5040/(1 x 720) = 7.

As you can see, because of the formula, the amount of 3-color combinations in a 7-color set is the same as the amount of 4-color combinations, i.e. 35.

Now as we have figured out the total amount of all possible unordered combinations the question is: what are unique schemes in which these combinations can be applied to paint the pyramid? It will be convenient to imagine we are painting our pyramids on the flat cardboard, as in the diagrams, before folding up.

If we take the 4-color subset (say, blue, green, red, and yellow), it can be applied in only 2 distinctive ways, as shown in Figs. 1 and 2. Any other way will only result in one of these when the pyramids are folded up. If we take three colors (say, green, red, and yellow), they may be applied in only 3 was shown in Figs. 3, 4, and 5. Two colors (say, green and yellow) may be also applied only in 3 ways shown in Figs. 6, 7, and 8. Any single color (say, blue) may obviously be applied in only 1 way shown in Fig. 9.

Multiplying the number of unordered combinations by the number of distinctive colored scheme for that combination we obtain the following amounts:
1) 4 colors: 2 x 35 = 70;
2) 3 colors: 3 x 35 = 105;
3) 2 colors: 3 x 21 = 63;
4) 1 color: 1 x 7 = 7.

Thus, a total amount of the unique ways how the pyramid me be painted in, using each time the colors from the solar spectrum is the sum of these four numbers: 70 + 105 + 63 + 7 = 245.