Using Puzzles to Teach Problem Solving

TEACHER'S GUIDE TO SHAPE BY SHAPE

Also includes Brick by Brick, Block by Block, and That-A-Way

BENEFITS

Shape by Shape is an assembly puzzle that is an exciting way to teach elements of

— problem solving
— visual and thinking
— geometry and measurement
— combinatorial reasoning

The game is self-directed, with 60 challenge cards that include hints. The activities in this guide will help your students get more out of Shape by Shape.

ABOUT THE GAME

Equipment. Shape by Shape includes 14 puzzle pieces (6 orange and 8 yellow), 60 challenge cards with answers on the backs, and a tray that doubles as storage for the cards.

How to Play. Select a puzzle card. The goal is to fit all the pieces in the tray to make a copy of the design. Every puzzle uses all 14 pieces.

Challenge Cards. On the back of each card are two hints: the solution to the orange area, and the solution to the yellow area. If you get stuck on a puzzle you can look at one of these hints without revealing the whole solution.

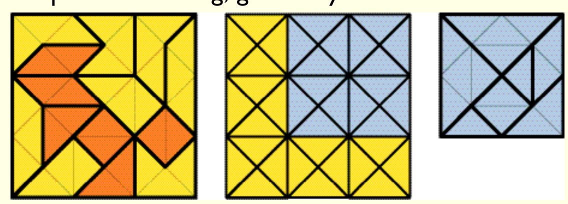
History. Shape by Shape was invented in 198x by Japanese puzzle designer Nob Yoshigahara.

RELATED GAMES

Brick by Brick features 5 pieces each composed of 3 bricks. Easier than Shape by Shape because single bricks are visible.
Block by Block features 7 pieces each composed of 3 or 4 cubes. Harder than Shape by Shape because pieces are 3D.
That-a-Way features 10 pieces each made of 2 arrows. Less about geometry. more about pattern.

Shape by Shape vs. Tangrams

Shape by Shape is closely related to Tangrams, a classic puzzle invented in China that became a world wide craze in the early 18th century. The seven simple Tangram shapes can be arranged to create thousands of different silhouettes, such as people, animals, letters, and everyday objects. Tangrams are used in schools to teach problem solving, geometry and fractions.



Shape by Shape and Tangrams are both based on the same triangular grid. They are similar enough that most Tangram activities work with Shape by Shape. Shape by Shape, however, is bigger and more complex. We recommend that you introduce math concepts with Tangrams first, then use Shape by Shape as a follow-up activity to explore concepts in more depth.

Shape by Shape is different enough from Tangrams that both are worth owning. The main differences are:

In a frame. The Shape by Shape pieces are intended to be assembled in a square frame, unlike Tangrams, which are assembled without a frame. The frame forces pieces to be in certain orientations and emphasizes the underlying grid more than in Tangrams. See page 8 *Seeing Grids* for details.

Two colors. Composing pictures using two colors of pieces within a frame opens up new artistic possibilities. See page 2 *Introducing Shape by Shape*.

More complex. Shape by Shape pieces have more complex shapes than Tangram pieces, so they are less suited to basic explorations in geometry. Because the areas of pieces are in the more complex ratio 3:2, rather than 4:2:1, Shape by Shape is less suited for teaching fractions. Shape by Shape is better for teaching combinatorics, however, since pieces include all combinations of 2 or 3 triangles. See page 15 *Counting Combinations* for details.

Pieces come in pairs. Shape by Shape pieces come in identical pairs, so unlike Tangrams you don't need two sets for 2-player games. See page 4 *Pack It in.*

Introducing Shape by Shape

Play the game, reflect on experience

Become familiar with the shapes.

— Pair up each piece with its twin.

— Count the sides. Which shapes have 3 sides? 4? 5?

— Name the shapes. Can you find a square, a triangle, a parallelogram, a trapezoid, a pentagon?

— Which shapes are convex and which shapes are concave? A shape is concave if it has at least one corner that points inward; otherwise it is convex.

— Find all the shapes that have right angles. Find all the shapes that have obtuse angles (an obtuse angle is bigger than a right angle).

Try it . Have students work through the puzzles on their own either individually or with a partner. Give each student a sheet to record their progress. You will find blackline masters on the next few pages. A few tips:

— Work through puzzles in sequence, or skip around.

— Start by assembling small isolated orange or yellow areas that divide into just one or two pieces. Then look for forced moves, as described at right.

— If all else fails, see the hints on the back of the card.

Reflect on experience . As with all manipulatives, students will get more out of Shape by Shape if they take time to reflect on their experiences.

— Write about your thought processes. What problem solving strategies did you try? Which worked best?

— Compare Shape by Shape with Tangrams. What is similar and what is different? Which do you find harder? Which makes better looking pictures?

— What artistic techniques do Shape by Shape pictures use that are not present in Tangrams? Can you find pictures that clip part of the picture at the edge of the frame, turn the frame 45° to make a diamond, or use the yellow area as the foreground instead of the orange? Which pictures have two or more orange areas that do not touch each other?

— What did you do when you got stuck? Explain through words or diagram the strategies you tried in solving a particular puzzle? It may be easier to record your thoughts if you talk about what you are thinking as you work on a puzzle, and let your partner take notes.

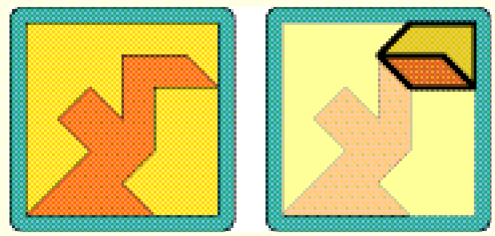
— What advice would you give to other students trying to solve Shape by Shape puzzles?

Forced mo ves

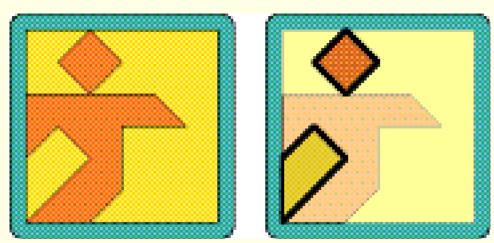
A good strategy for solving Shape by Shape puzzles is to look for forced moves — places in the figure where only one piece will fit. Often one forced move will unlock another forced move.

The principle here is to attack puzzles at their weakest point — which requires figuring where the weakest point might be. Finding forced moves is also related to the problem solving strategy of breaking problems into smaller subproblems.

The easiest forced moves to find are isolated orange or yellow areas that match just one piece, such as the head or the area below the left arm in challenge 9 “Sliding Man,” as shown below.



If there are not any small isolated areas, look for cul-de-sacs or peninsulas that force the choice of a particular piece.. For instance, the area in the corner above the head in challenge 11 “Flapping Bird,” forces the a particular yellow piece, and the head of the bird forces a particular orange piece, as shown below. Not all challenges have such clear forced moves.



Challenge cards for Shape by Shape, Brick by Brick, Block by Block, That-a-Way

Name: ___________________________

Puzzle	*Solved?*	*Comments, hints*
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Puzzle	*Solved?*	*Comments, hints*
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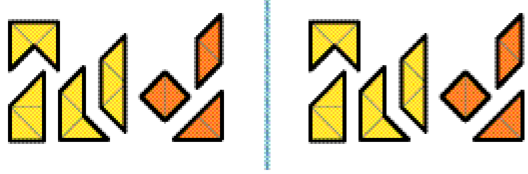
Pack It In

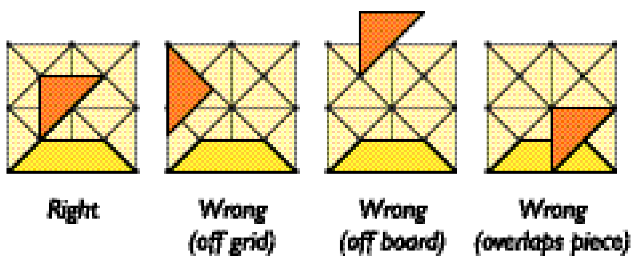
Game for 2 students, 1 set

This game for two players gets students familiar with the Shape by Shape pieces by exercising their ability to visualize how pieces fit together. It also exercises strategic thinking.

Setup. Each student gets half of the 14 pieces — one of each piece type.



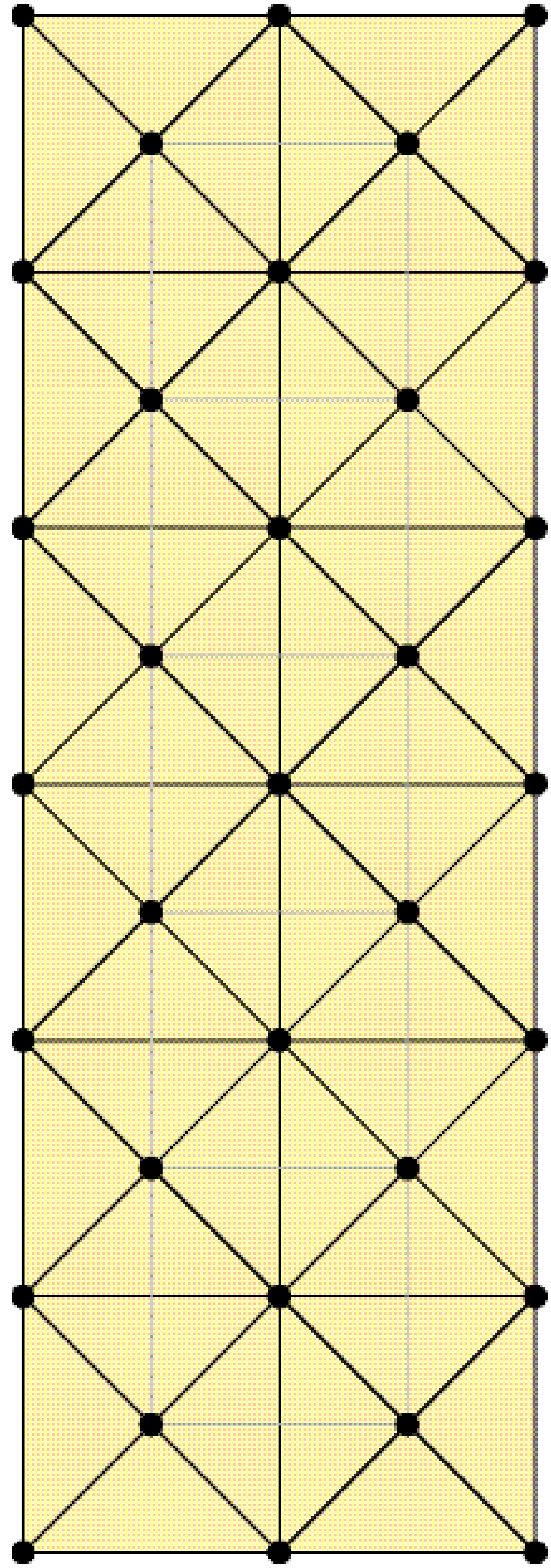
Activity. Students take turns placing pieces on the grid at right. Each piece must be placed so all corners touch black dots, and may not overlap other pieces or go off the edge of the board. For instance, here are correct and incorrect ways to place the orange triangle.



Tips. Try to place pieces where they block the other player from moving, but still allow you to move. Orange pieces are easier to place than yellow pieces, because they are smaller.

Reflection. What strategies did you develop in playing this game?

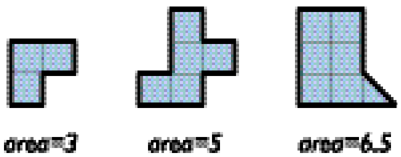
Going further. Try playing the same game with 2 sets of tangrams in 2 different colors. Each player gets a complete set of Tangrams. Again, the last player to place a piece wins. Because a double set of tangrams has slightly less area than a set of Shape by Shape pieces, you may want to shorten the board to 5 or 4 squares high.



MEASURING AREA

Activity for 1 student, 1 set

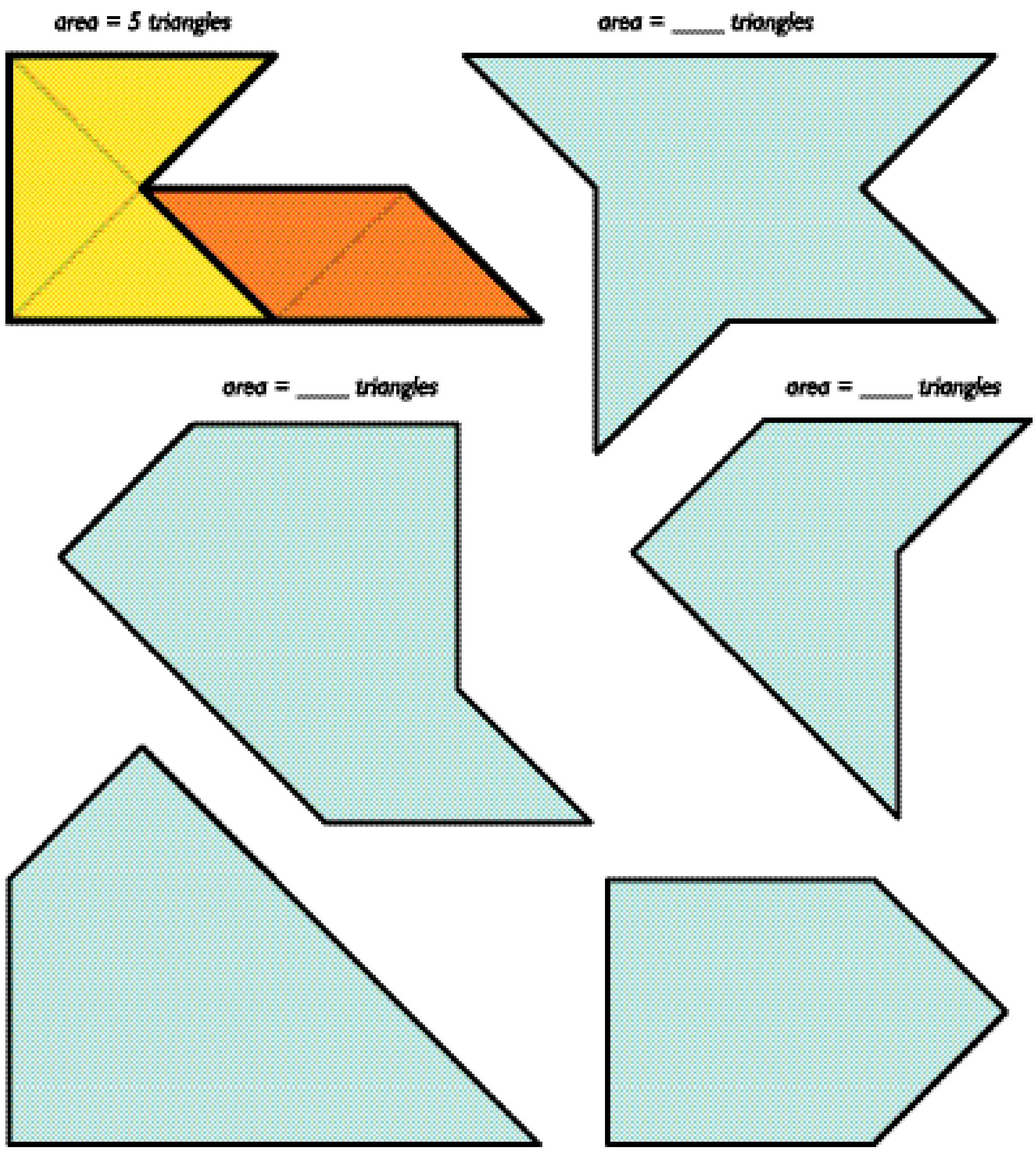
A good way to measure the area of a figure is to cut it into pieces that are all the same shape. For instance, the figures below have areas that are 3, 5, and 6 1/2 squares.



Similarly, we can measure the area of the Shape by Shape pieces by cutting them into right triangles.

Notice that the orange pieces all have an area of 2 triangles, and the yellow pieces all have an area of 3 triangles.

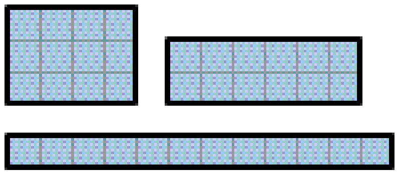
Measure the areas of the figures below by covering them with Shape by Shape pieces. Record the number of triangles in each figure? Which figure has the largest area? Which has the smallest? Which figures have equal area? Answers on page 21.



Measuring Perimeter

Activity for 1 student, 1 set

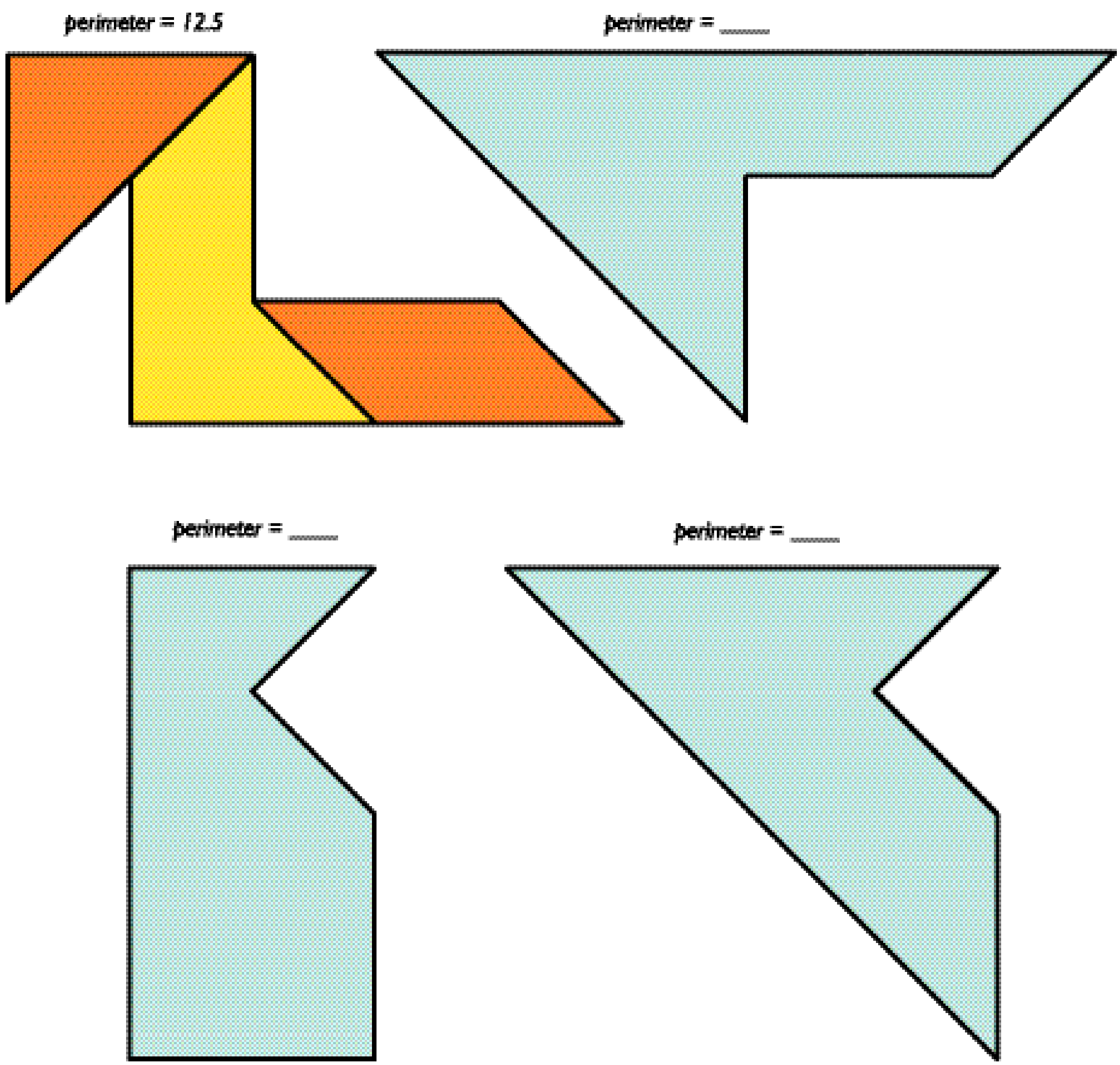
The perimeter of a figure is the sum of the lengths of all the edges. Figures that have thame area can have different perimeters. For instance, these rectangles all have area 12, but different perimeters. Which perimeter is longest?



The figures below were all made with the same three pieces so they all have the same area. First make each figure by covering it with the three pieces. Then measure the perimeter with a ruler. Some edges will be 1, 2, 3 or 4 inches long. Other edges will be slightly less than 1.5 or 3 inches; round these up to exactly 1.5 or 3 inches.

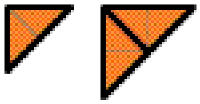
Record the perimeters in the blanks. Which figure has the longest perimeter? Which has the shortest? Can you make a figure with the same three pieces that has a shorter perimeter than any of these figures?

Answers on page 21.



Similar Figures
Activity for 1 student,

Two figures are similar if t
different sizes. For instanc



One way to check whethe
close one eye and hold th
figure. Move the small figu
the two shapes match exa

Below and at right is t
in four different sizes. Hol
to verify that all the shape
ure using Shape by Shape p

Write down the area
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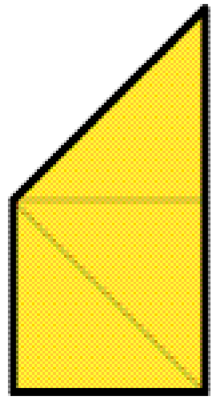
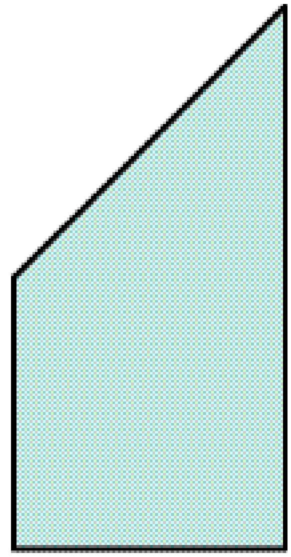
the area? When you multi
pens to the area? How ma
you need to make a simila
perimeter? Try making larg
Shape by Shape pieces.

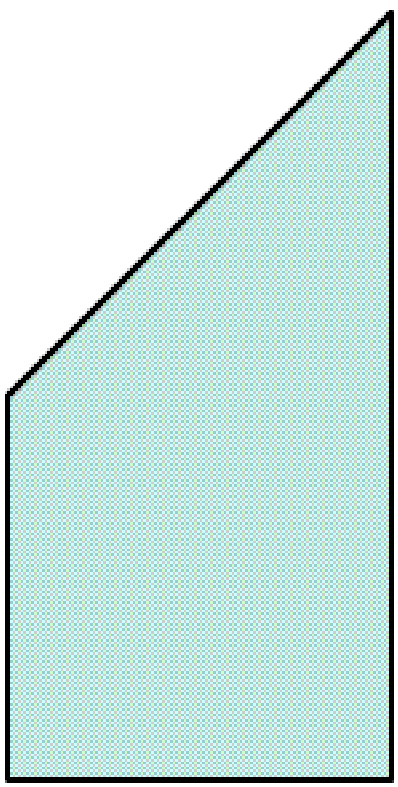
Answers on page 21.



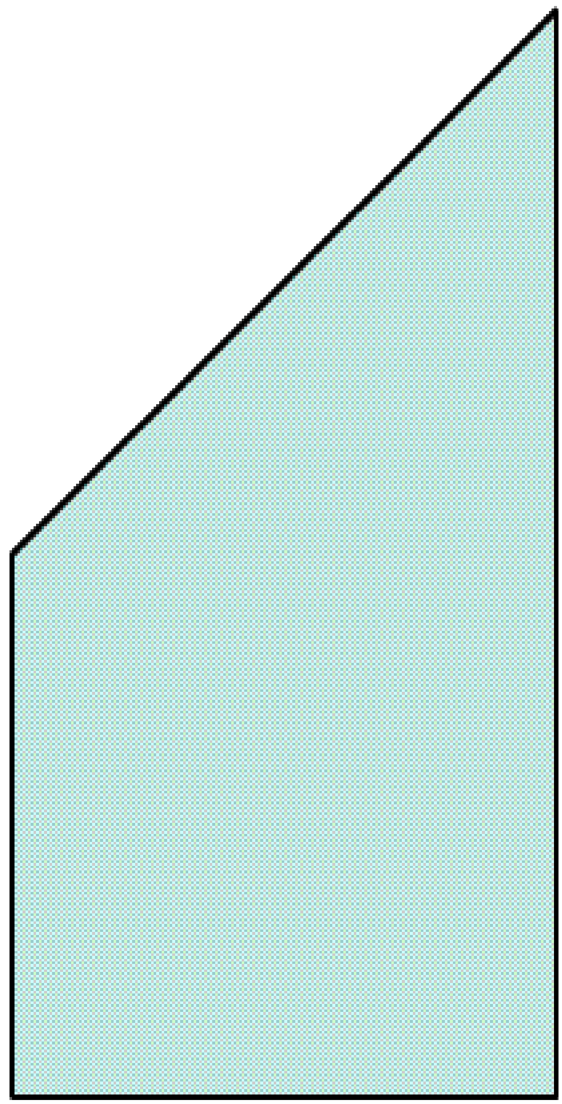
area = 3 triangles
perimeter = 5.5 inches



area = ____ triangles
perimeter = ____ inches



area = ____ triangles
perimeter = ____ inches

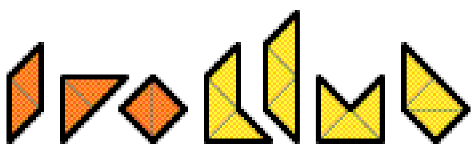


area = ____ triangles
perimeter = ____ inches

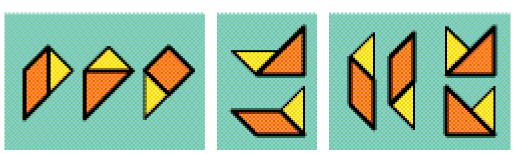
Seeing Grids
Perceptual activities

Every Shape by Shape piece is composed of the same identical component parts. To solve puzzles, it helps to visualize the components that make up each piece. This sort of visual regrouping is a key skill not only in geometry, but also in counting (chunking items into larger groups), algebra (grouping together related quantities), art (seeing scenes in terms of shapes), and reading (breaking words into syllables).

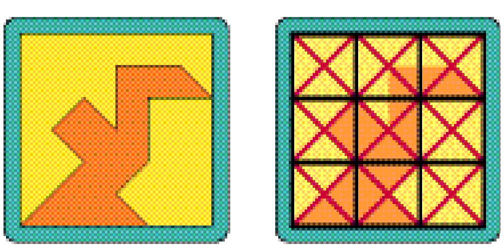
Parts . Hold each orange Shape by Shape piece and visualize the line dividing it into two congruent isosceles right triangles. Tip: trace the line with your finger, or draw a sketch. For each yellow piece, visualize the lines dividing it into three congruent isosceles right triangles.



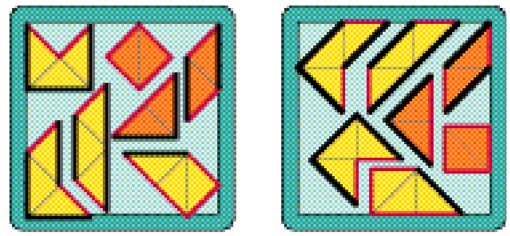
Orange within yellow . Hold up each yellow piece and in your mind split it into an orange piece and a leftover isosceles right triangle. Tip: trace the line with your finger, or draw a sketch. Which yellow piece(s) contain three different orange pieces? Two? One?



Grid . The big difference between Shape by Shape and Tangrams is that pieces are assembled in a frame, which forces pieces to align with a grid as shown below. Choose a Shape by Shape card and draw the figure on paper. Can you draw the grid on the design? Every corner of the design should hit a corner of the grid. Can you imagine the grid without drawing it?



Piece orientation . Here are the proper orientations of Shape by Shape pieces. Pieces may be turned as long as the edges highlighted in red remain horizontal or vertical. If you turn pieces so the red edges run diagonally, they will never fit in a finished Shape by Shape figure. This means 90° rotations are okay, but 45° rotations are not.



Correct orientation   Wrong orientation

It is easy to see that the correct orientations align with the grid. But what is wrong with the other orientations?

Measure the pieces and you will find there are two types of edges. Red edges, which run diagonally, come in multiples of 1 inch, and black edges, which run horizontally and vertically, come in multiples of 1.4 inches. The frame is 3 times 1.4 inches, or about 4.2 inches on a side.

The problem with turning a piece 45° is that the two types of edges are incompatible. For instance, if we put an orange square in a corner of the frame, the remaining length along a side of the frame is 3.2 inches. But no combination of 1 and 1.4 inches adds up to 3.2 inches. Siimilarly, the only way to make change for 42 cents with 10- and 14-cent coins is with three 14-cent coins.

For students familiar with the Pythagorean theorem there is a deeper reason. If a red edge is 1 inch, then the black edge has length $\sqrt{2}$, or about 1.4142... Repeatedly adding up an irrational number never yields an integer.

Applications of Grids . Discuss ways people in many areas of art and science use grids to help them organize their thinking. Here are some examples.

— Mathematicians and cartographers use cartesian coordinate grids to locate points in space.

— Artists use grids to help them see proportions accurately. Architects build houses on a grid in which vertical beams are spaced 18 inches apart.

— Computer games often use grids to divide space. For instance, The Sims uses a 4 by 4 foot square module.

— Engineering. Computer chips are laid out on grids with a spacing that is measured in hundreds of atoms.

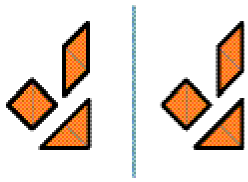
— Graphic design. Magazine designers use grids to help them place titles, text and illustrations on a page.

Copy My Shape

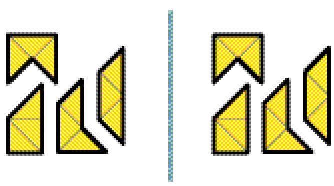
(2 students, 1 set, pencil and paper)

As with tangrams, it is easy for students to create Shape by Shape challenges by arranging shapes and tracing the outline. In this game players create puzzles that do not use the board.

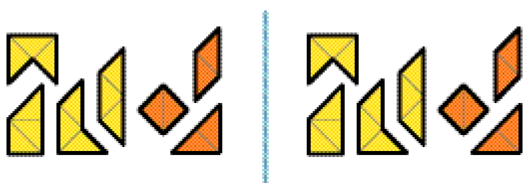
Setup. Each student needs a pencil and several sheets of paper. Give each player the identical assortment of pieces. For an easy game appropriate for younger ages, divide the 6 orange pieces into two identical sets of 3.



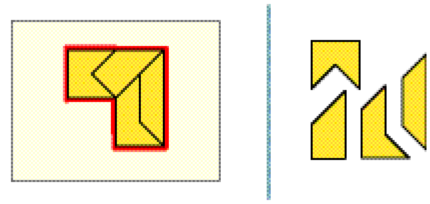
For an intermediate game, appropriate for most ages, divide the 8 yellow pieces into two identical sets of 4.



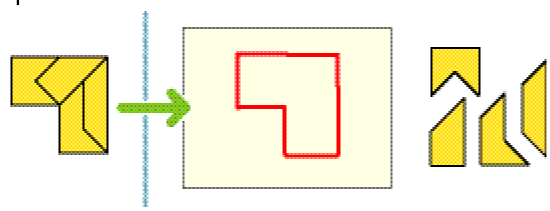
For an advanced game, appropriate for older students, divide all 14 pieces into two identical sets of 7.



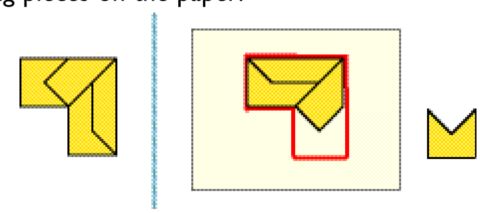
Gameplay . The first student secretly assembles a shape by arranging all the pieces on a piece of paper, without letting the other student see it, then carefully outlines the shape by drawing on paper. Since pieces are arranged without a board, they do not have to conform to a grid, and can be turned at any angle.



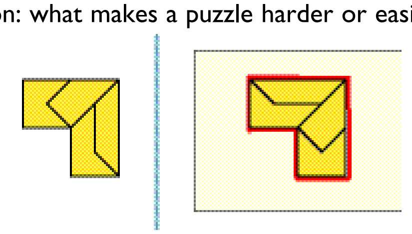
The first student slides the paper out without disrupting the pieces, and gives it to the other student, keeping the pieces hidden.



The other student now tries to fill in the outline by placing pieces on the paper.



Compare the two solutions and see if they are the same. In this case the second solution is the same as the first, reflected about a diagonal line. Take turns making up puzzles for each other to solve. Question for discussion: what makes a puzzle harder or easier?



Ambidextrous Symmetry

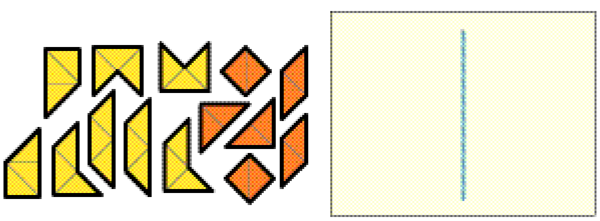
Activities for 1 student, 1 board

In these activities students experience symmetry in their bodies by manipulating Shape by Shape pieces. You can also do this exercise with double tangram sets or with pattern blocks, as long as there are two copies of each shape. Understanding an idea in your body is quite different from understanding it in your mind, so even advanced students will find this activity challenging.

Ambidextrous Reflection (1 student, 1 set). Warm up by pretending to conduct an orchestra by moving your left and right hands in the air at the same time in opposite directions. Be sure your hands maintain perfect mirror symmetry if the left hand moves left, the right hand should move right and vice versa.

Try different rhythmic motions: apart and together, circles in opposite directions, figure 8's. Try twisting your hands, bending your fingers, waving your elbows. Focus on experiencing the sensation of moving your hands symmetrically. It may feel a bit like swimming.

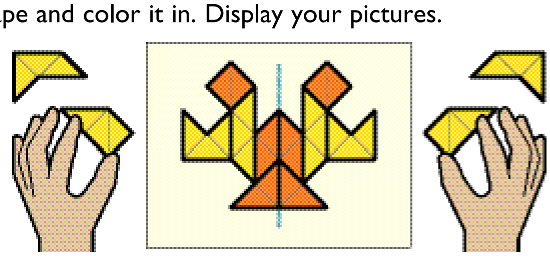
Draw a vertical line in the middle of a piece of paper. Dump the Shape by Shape pieces near the paper.



With both hands moving simultaneously, grab two identical pieces. Position the pieces so they are on opposite sides of the line and at equal distances. Turn the pieces until they are perfect mirror images of each other. You may have to flip pieces over.

Now with both hands simultaneously, move both pieces in perfect mirror symmetry. Try moving the pieces together and apart, turning the pieces, and tracing different motions. Enjoy the sensation of manipulating pieces symmetrically.

Bring the pieces together until they meet at the center. Keep assembling pieces until all are used. The shape will automatically have mirror symmetry. Trace the final shape and color it in. Display your pictures.

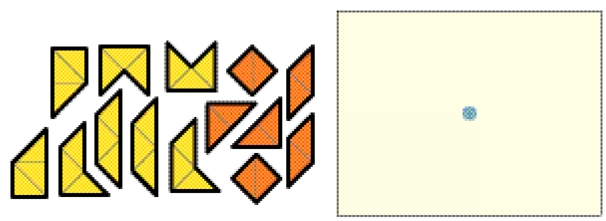


Ambidextrous Rotation (1 student, 1 set). Warm up by grabbing an imaginary steering wheel with both hands. Turn the wheel left and right, pretending to drive. Make the steering wheel bigger and smaller so your hands move apart and together. Be sure your hands maintain rotational symmetry: they should always be on opposite sides of the steering wheels and at equal distances from the center.

Alternatively, students can visualize stretching a rubber band between their two hands, trying to keep a dot in the middle of the band motionless in space. Most students find that moving in rotational symmetry is much harder than moving in reflective symmetry.

Open your hands so your palms face each other. Try twisting your wrists both counterclockwise or both clockwise (twisting in opposite directions breaks the symmetry). Focus on the sensation of moving your hands with rotational symmetry. It may feel like two people dancing.

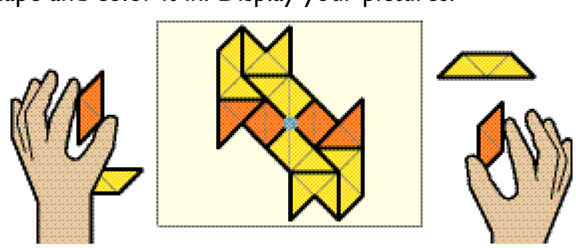
Draw a dot in the middle of a new piece of paper. Dump the Shape by Shape pieces near the paper.



With both hands moving simultaneously, grab two identical pieces. Position the pieces so they are on opposite sides of the dot and at equal distances. Turn the pieces until they are perfect 180 degree rotational images of each other. You may have to flip pieces over.

Now with both hands simultaneously, move both pieces in perfect rotational symmetry. Try moving the pieces together and apart, turning the pieces, and tracing different motions. Enjoy the sensation of manipulating pieces symmetrically.

Bring the pieces together until they meet at the center. Keep assembling pieces until all are used. The shape will automatically have mirror symmetry. Trace the final shape and color it in. Display your pictures.



Follow the Leader

Activities and game for 2 students, 1 board

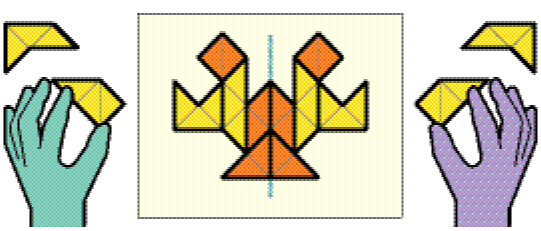
Here are two-person variations on the ambidextrous symmetry exercises. Instead of one person manipulating two pieces with two hands, two people manipulate two pieces each using one hand. Moving symmetrically with two people instead of one is harder and requires close cooperation. Once students start this exercise they may not want to stop.

This activity requires space for students to stand up and move. Push desks to the edge of the classroom or move to a larger space. You can do this activity without moving desks, but students will need to move more carefully so they don't bump into each other.

Mirror the Leader (2 students). Warm up by standing up facing each other. One person is the leader and the other is the follower. The leader slowly moves a hand and the follower moves the opposite hand as if looking in a mirror. Try moving your arms, shifting your legs, and twisting your body. Change who leads and who follows. What happens if you try to shake hands?

As in Ambidextrous Reflection, draw a vertical line down the middle of a piece of paper. Divide the Shape by Shape pieces into a two identical heaps of 7 pieces, one on each side of the line.

One student sits in front of each heap. Again one student leads and the other follows. The leader slowly picks up a piece and the follower picks up the identical piece from the other heap. Turn both pieces so they are perfect mirror images of each other. Slowly move the pieces, being sure to maintain symmetry. Assemble the pieces in the middle to make a symmetrical figure. From time to time switch who leads and who follows.

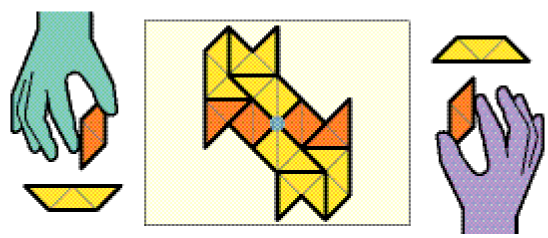


Rotate the Leader (2 students). Warm up by standing up facing each other a few feet apart. One person is the leader and the other is the follower. Pretend there is a tall tree growing in the spot exactly half way between you two. The leader slowly starts walking around the tree, and the follower slowly moves in symmetry, always staying on the opposite side of the tree and at the same distance.

Try moving your arms and bending your body. Be sure to maintain rotational symmetry, not reflective symmetry: if the leader raises a right hand, the follower should also raise a right hand, not a left hand. Change who leads and who follows. What happens if you try to shake hands?

As in Ambidextrous Rotation, draw a dot in the middle of a piece of paper. Divide the Shape by Shape pieces into a two identical heaps of 7 pieces, one on each side of the dot.

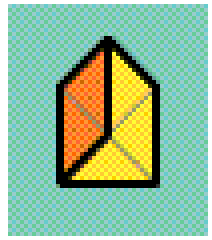
One student sits in front of each heap. Again one student leads and the other follows. The leader slowly picks up a piece and the follower picks up the identical piece from the other heap. Turn both pieces so they are perfect rotational images of each other. Slowly move the pieces, being sure to maintain symmetry. The two pieces should always be on opposite sides of the center, and at equal distances. Assemble the pieces in the middle to make a symmetrical figure. From time to time switch who leads and who follows.

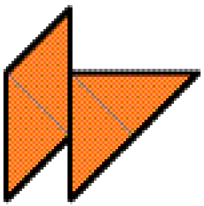


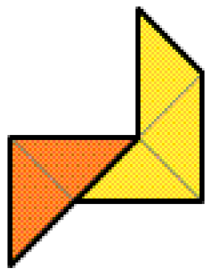
Symmetric Pairs

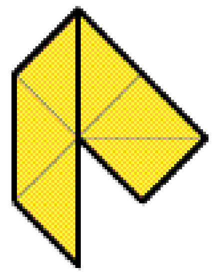
Activities and game for 2 students on 1 board

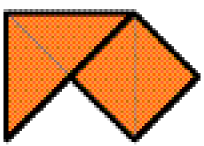
At right is a symmetrical figure composed of two asymmetrical pieces. The figures below are not symmetrical. Can you rearrange each of the pairs of pieces into a symmetrical shape? One pair cannot be assembled into a symmetrical figure. Can you figure out which? Can you find other pairs of Shape by Shape pieces that can be assembled into symmetrical figures? Answers on page 21.

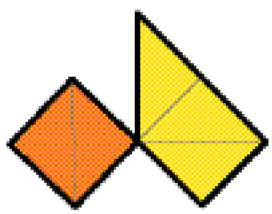


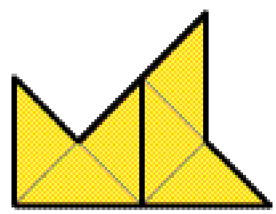
1. 

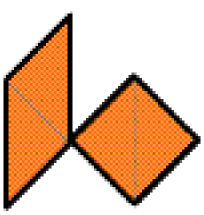
2. 

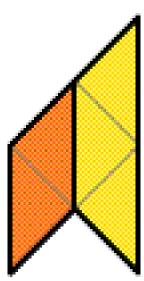
3. 

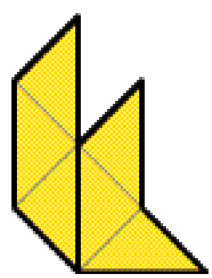
4. 

5. 

6. 

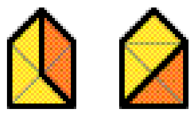
7. 

8. 

9. 

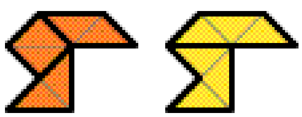
SUBSTITUTION
Activities for students working in pairs

3+2 = 2+3 . Make a shape with one orange and one yellow piece. Can you make the same shape a different way with one orange and one yellow piece? Some of the pieces can be the same, but they must be in different positions. If not, make a different shape and try again, or choose a different pair of yellow pieces. Here is one solution. How many other sollutions can you find?

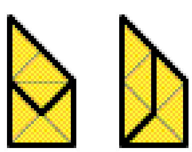


Hint: imagine subdividing pieces into triangles. Be sure both pieces align to the same grid, or there will be no solution.

3x2 = 2x3 . Make a shape with two yellow pieces. Can you make the same shape with three orange pieces? If not, make a different shape with the yellow pieces and try again, or choose a different pair of yellow pieces. Hint: imagine the subdivision into triangles. Here is one solution. How many other solutions can you find?



3+3 = 3+3 . Make a shape with two yellow pieces. Can you make the same shape with a different pair of yellow pieces? If not, make a different shape with the first two pieces and try again, or choose a different pair of pieces. Here is one solution. How many other solutions can you find?



6 orange = ? . Here is a figure made of the 6 orange pieces and no yellow pieces. The same figure can also be made with other numbers of orange and yellow pieces. Without touching any actual pieces, can you predict what other numbers of orange and yellow pieces might work? Can you find actual solutions that match those numbers? Answers on page 21.

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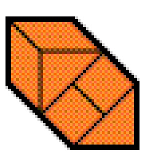
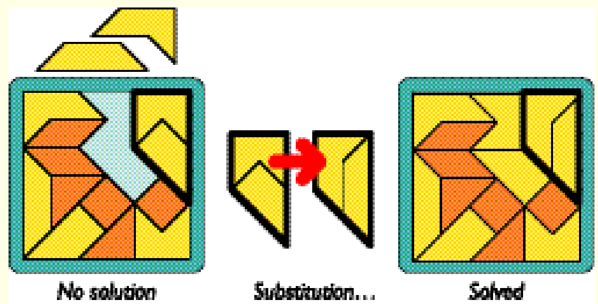


Substitution as a problem solving strategy

In solving a polyform puzzles you will often find that you need a piece that has already been placed on the board. Sometimes you can free up a piece by substituting a different pair of pieces for a shape within the solution. For instance, the puzzle at left cannot be completed, but by substituing pieces in the outlined area you can solve the puzzle.



Eschanging subgroups is also a good way to generate alternate solutions to polyform puzzles.

Other applications of substition . Being able to mentally substitute one subdivision for another is a useful skill in many fields. Where can you find regrouping being used in the world around you? Here are a few examples.

— Algebra. One of the most common operations in algebra is substituting a number for a variable. Another important part of algebra is becoming facile with common substitutions, such as substituting $(x+1)(x-1)$ for x^2-1.

— Cooking. A good cook can substitute other ingredients for missing ingredients.

— Writing. Fluent writers can reword a thought many different ways.

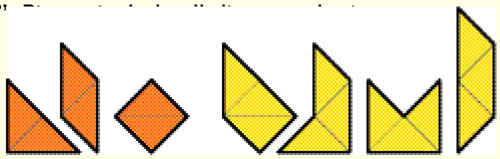
— Music. One of the richest applications of substitution is in rhythm. For instance, a group of six beats can be divided up as two groups of three beats or three groups of two beats. Mathematically, 2x3 = 3x2. Have two students clap even beats in unison. Then have one student clap louder on every third beat, and the other student clap louder on every second beat. The two patterns will coincide every six beats. Visually, this pattern looks like:

1 2 3 1 2 3 1 2 3 1 2 3 1 2 3 1 2 3 1…

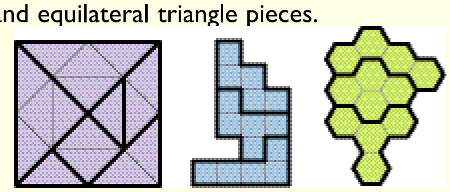
1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1 2 1…

About Polyforms

Shape by shape is a polyform puzzle — every piece is composed of several copies of a single module. In Shape by Shape the module is an isosceles right triangle, also called a “tan”



Other polyform puzzles include tangrams (1, 2 or 4 right triangles), pentominoes (5 squares), and tetrahexes (4 hexagons). By the way, pattern blocks are *not* polyforms, since there is no single module that can make up both the square and equilateral triangle pieces.

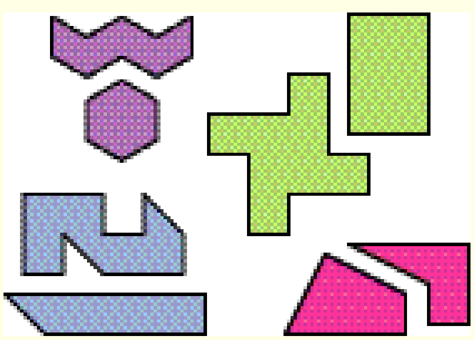


Here are prefixes and root words for other polyforms:

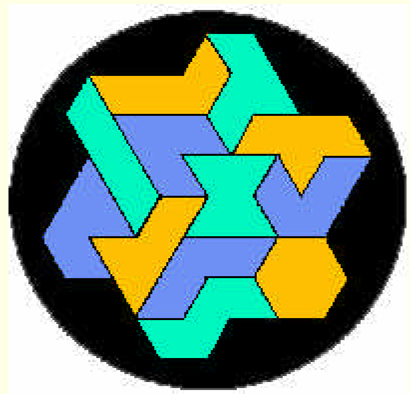
Prefix	*Root*
di (2)	**omino** (square)
tri (3)	**iamond** (equilateral triangle)
tetra (4)	**hex** (equilateral hexagon)
penta (5)	**tan** (isosceles right triangle)
hexa (6)	**cube** (as in Block by Block)
hepta (7)	**stick** (line segment on square grid)
octa (8)	**brick** (1x4 rectangle in brick pattern)

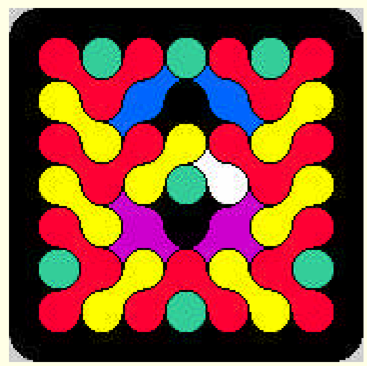
Polyominoes were named by USC mathematician Solomon Golomb, who whimsically reasoned that if a domino is made of two squares, then a tromino must be made of three squares, a tetromino four squares, and so. Similarly, a diamond is made of two triangles, a triamond three, etc.

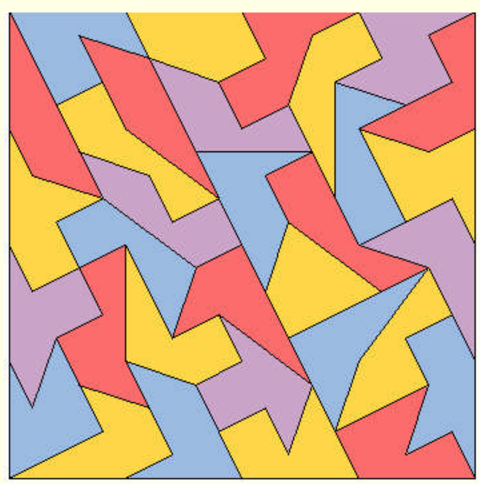
Can you deduce the modules in these polyforms? In each pair one figure is composed of 3 modules and the other is composed of 4. Answers on page 21.



Here are some of the beautiful polyform patterns that people have discovered. Most polyform puzzles are created by finding all polyforms for a given number of a particular module, then finding a pleasingly symmetrical way to fit them together. Aesthetically, polyform puzzles embody a balance between order and disorder. These examples are from gamepuzzles.com and mathpuzzle.com.







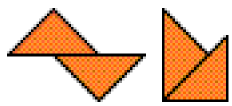
Counting Combinations

Activity for students working in pairs

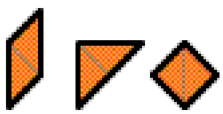
Polyforms make a wonderfully accessible introduction to combinatorics, a leading area of modern mathematics.

Counting polytans (2 students, 4 tans). Each group will need 4 isosceles triangles, all the same size. We will call these shapes "tans." You can cut tans out of paper, or use pieces from a tangram or Shape by Shape set.

Without looking at Shape by Shape, draw all the different shapes that can be formed by joining two tans (ditans). If students work in pairs, one can record while the other moves pieces. Tans must join along the full length of an edge, so these shapes are not allowed:

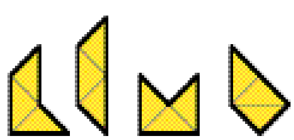


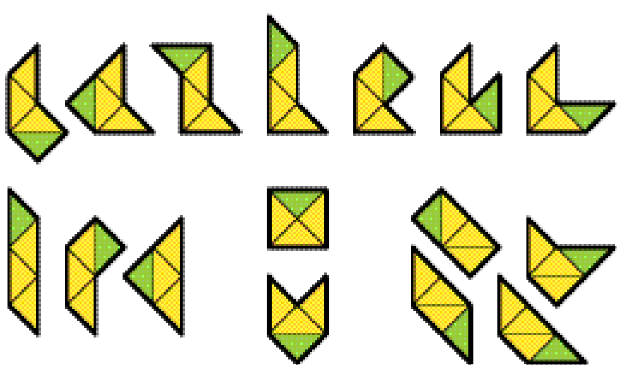
You will find there are just three such shapes — the shapes of the three orange pieces in Shape by Shape.



Issue for discussion: If we flip the pieces over on a table, the triangle and square are the same, but not the parallelogram. Why? If we choose to count flipped pieces as different, then there are four possible ditans, not three.

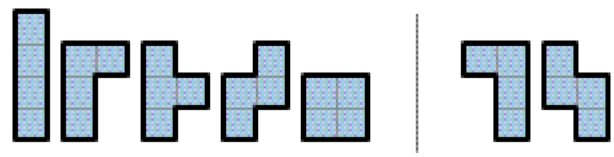
Count the number of possible tritans and tetratans. You will find that the possible tritans exactly match the yellow pieces in Shape by Shape. Older students may also want to count the number of pentatans, which is much harder. Here are all possible tritans and tetratans. I enumerated the tetratans by adding a triangle to the tritans in every possible position, then eliminating duplicates.





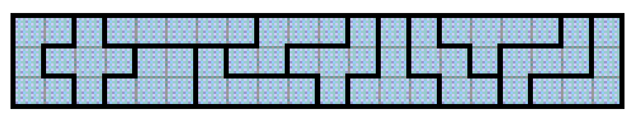
When students are done, have them compare solutions and see if their answers agree. Ask students to describe the methods they used to make sure every shape was included. For instance, one way to count pentatans, shown above, is to list all the tetratans (shown in yellow), then add an extra tan (shown in green) in every possible place around every tetratan, carefully omitting duplicates.

Counting polyominoes (2 students, 5 squares). There are two different trominos — a bar and an L. How many different tetrominos are there? Not counting mirror images, there are 5 different tetrominoes. The computer game Tetris uses 7 tetrominoes, including mirror images.

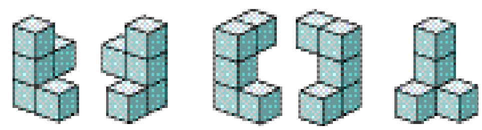


The 5 tetrominoes... and 2 mirror images

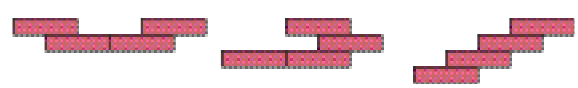
Counting pentominoes is trickier, and makes a nice 20-minute challenge. The pentomino counting problem appears in Arthur C. Clarke's novel Imperial Earth. There are 12 different pentominoes, which can pack a 3x20, 4x15, 5x12 or 6x10 rectangle. Hexominoes are much harder; there are 35.



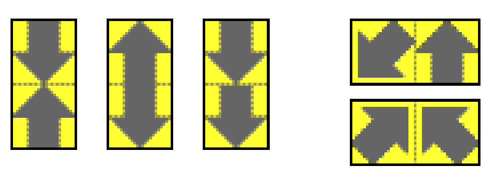
Block by Block uses shapes made of 3 or 4 cubes. Are these all of the tricube and tetracubes? If not, what shapes are missing? (Answer: 3 or 4 cubes in a line, or 4 in a square.) How many pentacubes exist? (Answer: 30, counting mirror images as different). Here are some pentacubes.



Brick by Brick uses tribricks. How many tetrabricks are possible? (Answer: 16). Here are some tetrabricks.



That-a-Way uses dominoes composed of two arrows. Each arrow can point in four directions. List the 10 "diarrow" pieces. If arrows can point only in or out there are 3 diarrows. If arrows can point in 8 directions, there are 36.



CREATE YOUR OWN

Open-ended creative activities

Problem posing is an important part of problem solving, since in real life there is no teacher to tell you which problems to solve. These activities that involve students in creating their own Shape by Shape challenges.

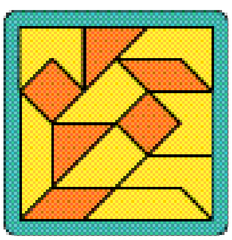
Shape by Shape. (1 student, 1 set). Invent a Shape by Shape puzzle by fitting the pieces into the square frame to form a pleasing picture. Give your puzzle a name and record it on the challenges sheet shown on the next page.

Give your puzzles to other students and watch them try to solve it. Was it easier or harder than you expected? Where did the other students get stuck? Did the other students find a different solution than yours?

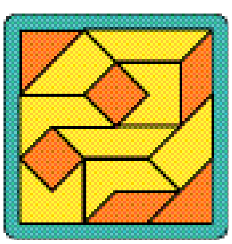
A good way to construct a puzzle is to arrange the orange pieces, then try to fill in the remaining area with yellow pieces. This can be quite hard, and does not always work. As with tangrams, most of the challenge is aesthetic: can you make a meaningful picture.

Construction challenges. Here are some difficult challenges for advanced students.

— Construct a puzzle in which no two orange pieces touch each other along an edge. Touching at corners is okay. Here is one solution:



— Construct a puzzle with as many completely disconnected clumps as orange pieces as possible. Here is a solution with 5 clumps. Probably 6 disconnected clumps is impossible.



— Construct a puzzle with as many disconnected yellow regions as possible. Two pieces must touch along an edge, not just at a corner, to be considered connected. Is it possible to get 8 disconnected regions? Answer: no, but 5 is possible, as shown above.

— Construct a puzzle that has a unique solution.

— Construct a puzzle with as many solutions as possible.

— Construct an image in which the yellow area, not the orange arrow, forms a meaningful figure.

— Pose other construction challenges and see if you can solve them.

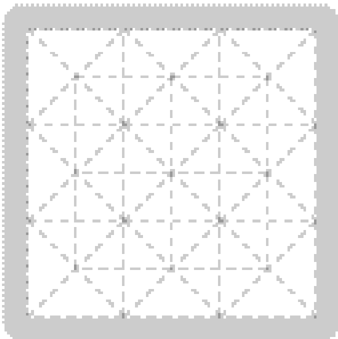
Make your own pieces. print out the pieces on page 17 on heavy paper and cut them out. These pieces are the same size as the plastic Shape by Shape pieces. What can you do with more than one set?

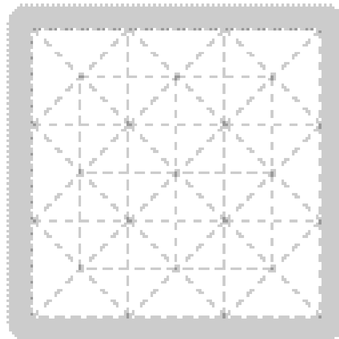
Change the shapes. Invent your own polyform puzzles. What shapes can you make with the 16 pentatans? Draw your puzzle by hand or by computer on heavy paper and cut it out.

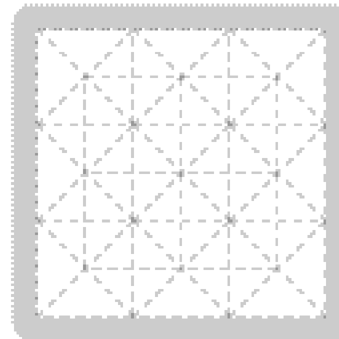
Changing the rules. Invent a new game based on Shape by Shape. Try combining aspects of Shape by Shape, Tangrams, Block by Block, Brick by Brick, and That-a-Way.

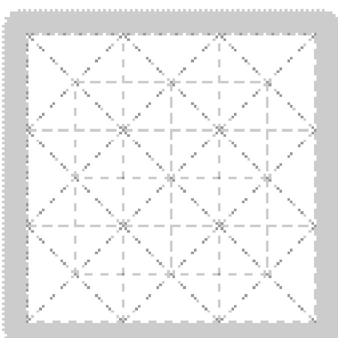
Programming. Suppose you wrote a program to solve Shape by Shape puzzles. Can you describe how it would work? How could you be sure you had found all the solutions?

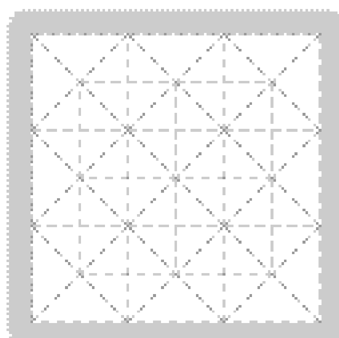
Name: ______________________

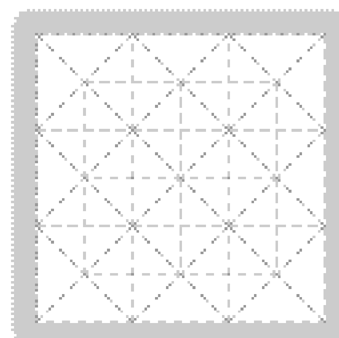


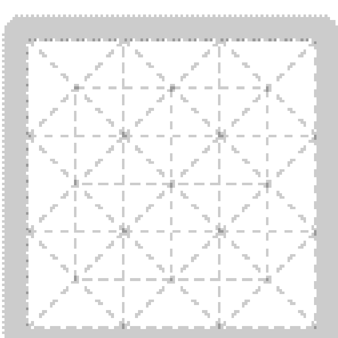


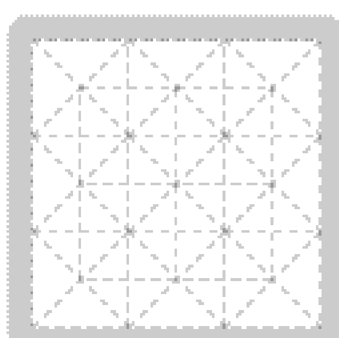


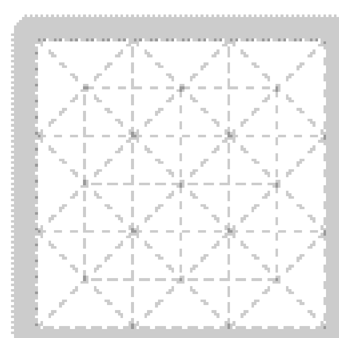






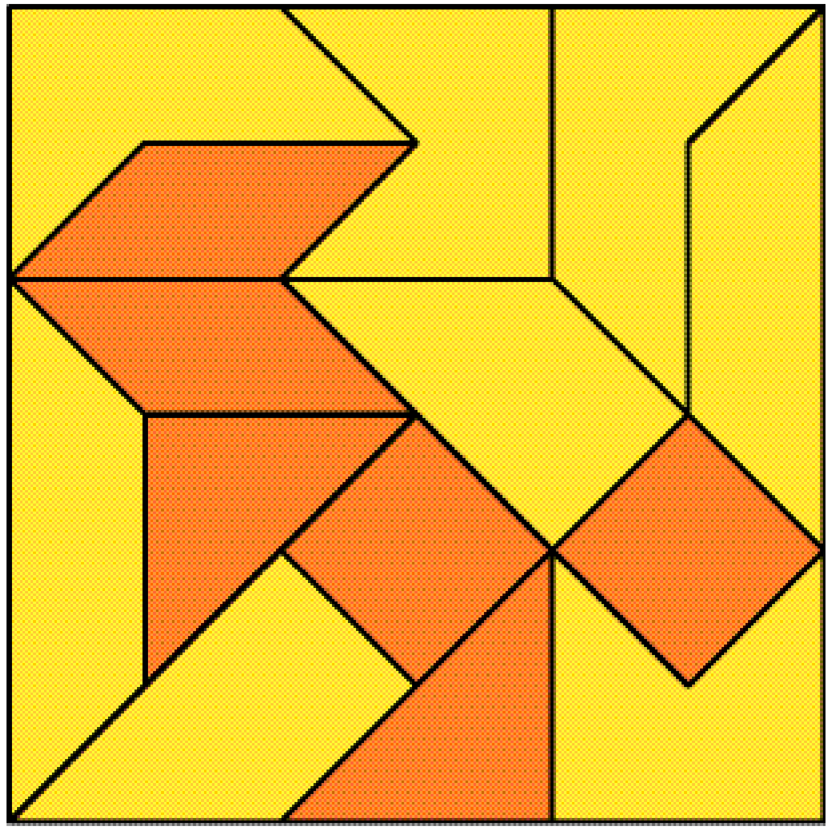


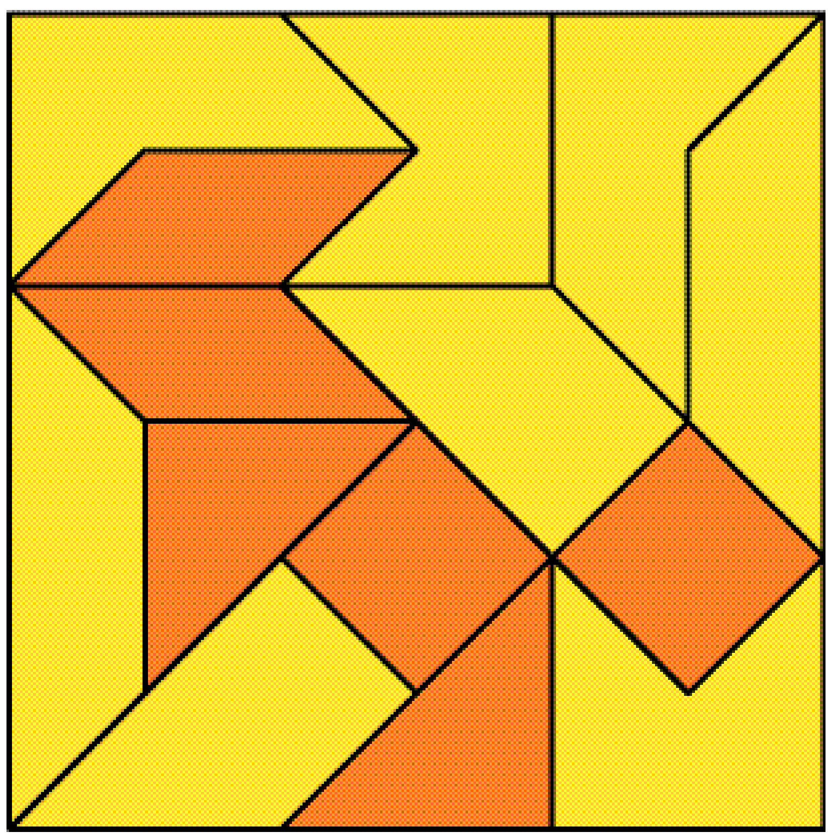




Make Your Own Set

Make your own pieces . print out the pieces on page 17 on heavy paper and cut them out. These pieces are the same size as the plastic Shape by Shape pieces. What can you do with more than one set?

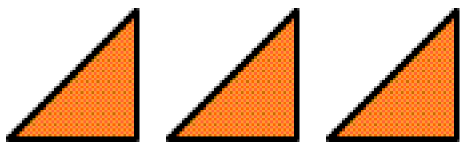




TRIANGLE DANCE
Kinesthetic activity for groups of 3 students

This activity lets you experience the shapes of Shape by Shape with your whole body. This is as much an exercise in creative movement as it is an exercise in geometry. For more whole-body math-dance activities, see the Math Dance web site at mathdance.org

Setup. You will need three isosceles right triangles of the same size, in which the shorter edges are about a foot long. The triangles should be light and rigid. The best alternative is to have shapes cut out of 2 inch thick furniture foam by a foam shop. You can also cut triangles out of cardboard. Triangles with thick edges are better because they let pieces meet easily at edges without overlapping.



Clear space to allow students to stand up and move by pushing desks to the side of the classroom or moving to a larger space. If you prefer to have students work sitting down, use smaller triangles such as those found in plastic tangram sets, with a side length of 2 to 6 inches. Students can then slide triangles on their desks rather than holding them in the air.

Activity. Your goal is to move the triangles so they form all seven of the Shape by Shape pieces in succession. You choose the order. Each student in a group holds one triangle. Of course some of the piece shapes will leave out one of the triangles.

First improvise a solution, then practice the sequence of movements until you can do it smoothly without hesitation and without talking. Perform your triangle dance for rest of the class. Adding music can help make the activity feel more like a performance. The best music for this purpose has no words, and is forward-moving but relaxed, such as instrumental jazz or classical music.

Moving two triangles together to form a square. These squares were cut out of furniture foam, with cylindrical handles screwed to masonite squares glued to the middles of the backs.

Tips. Hold your triangles so they lie in a vertical plane, with the front faces pointed toward the audience. Keep them in a vertical plane as you move them. Practice moving smoothly and precisely.

Pay special attention to starts and stops, so the compound shapes are clearly emphasized. How you hold the pieces is part of the performance, so don't fidget; keep the audience's focus on the shapes.

Experiment with different ways to form and unform the piece shapes. Try gradual and sudden, slow and fast, relaxed and tense, straight and circular movements.

Reflection. How was manipulating the big triangles in space different from working with small triangles on a table. What was harder and what was easier? What challenges came up in coordinating your group.

Going further . If you have animation software such as Geometer's Sketchpad or Macromedia Flash, you can try making a triangle dance movie. Choose music to for your soundtrack. Explore more than three triangles, triangles of different colors, and shapes other than triangles.

Learning from Shape by Shape

Big open-ended discussion questions

Visual thinking. What are some of the visual imagination skills that Shape by Shape exercises? Where else might these skills be useful?

Comparisons with other games. Compare Shape by Shape to Tangrams, Brick by Brick, Block by Block, and That-a-Way by answering the following questions .

— How is it similar to or different from Shape by Shape?

— Do you find it easier or harder? Why?

— Are problem solving strategies similar or different?

— Can you imagine a new game that blends this game with Shape by Shape?

Apply strategies to other situations. Applying what you have learned to other situations is one of the best ways to make it your own.

— Mathematics. Discuss connections between Shape by Shape puzzles and solving problems in mathematics.

— Decorative arts. Fabrics and wallpaper often use repeating patterns. Make a visually pleasing pattern using Shape by Shape pieces.

— Packing. How do problem solving strategies from Shape by Shape apply to packing a suitcase?

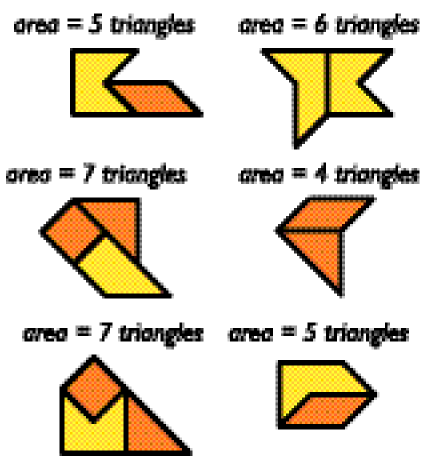
Classifying puzzles. These discussion questions further develop a critical understanding of problem solviing skills.

— Compare puzzles. Which puzzles are best for developing which types of problem solving skills?

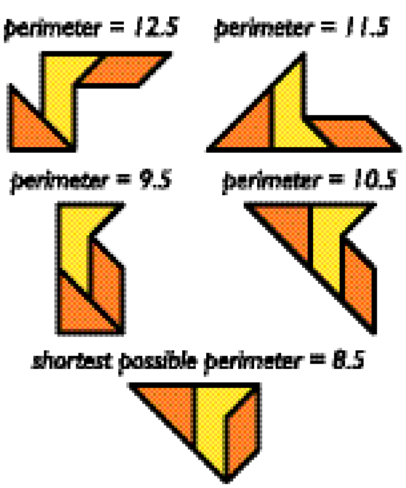
— Classify puzzles using Venn diagrams. What are the attributes for classifying puzzles? Invent new types of puzzles for categories with no members.

ANSWERS

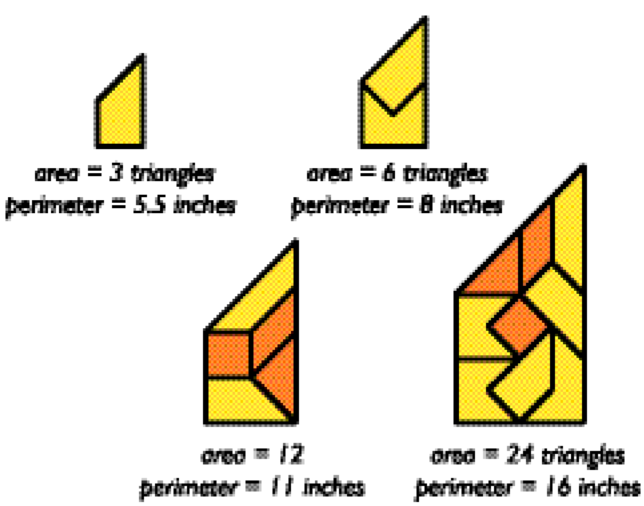
Answers to Measuring Area (page 5)



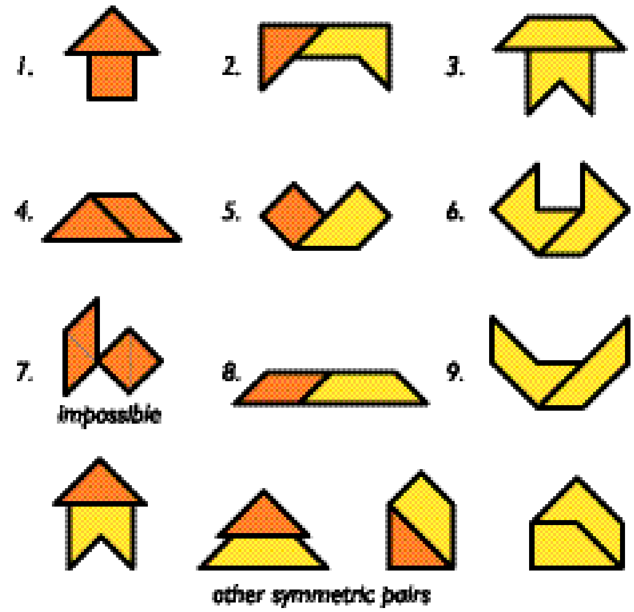
Answers to Measuring Perimeter (page 6)



Answers to Similar Figures (page 7)

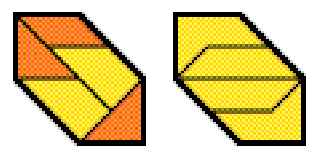


Answers to Symmetric Pairs (page 12)



Answers to 6 orange = ? (page 13)

3 orange and 2 yellow, or 4 yellow:



Answers to Polyform Grids (page 14)