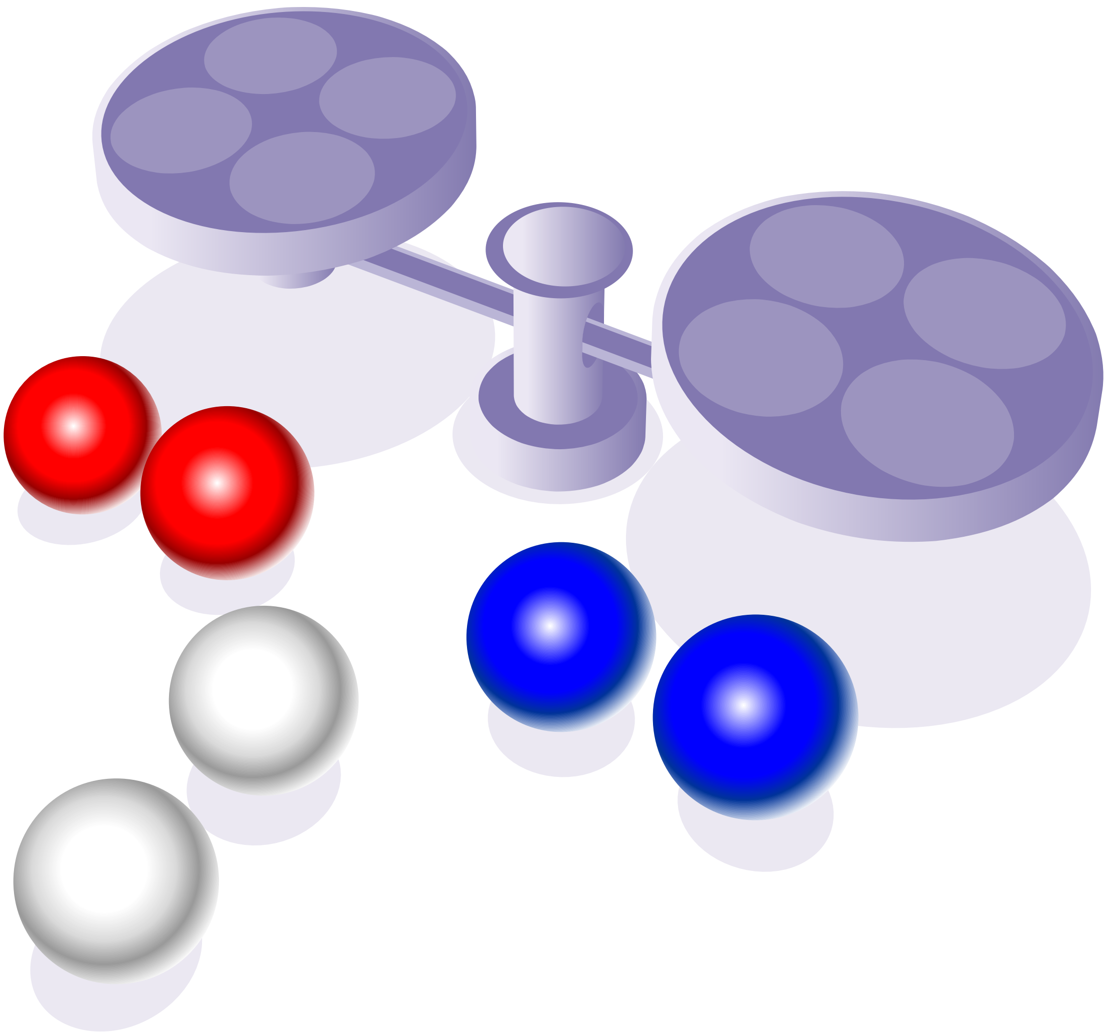

There are three pairs of balls - red, white, and blue. In each pair one ball is a little bit heavier than another one. All the heavy balls weigh the same, and all the light balls weigh the same. Also you have a balance scale.

Now, in just two weighings you have to determine the light and the heavy balls in each pair. How can it be done?

This was a really hard and tricky challenge, and we got a lot of different solutions to this clever puzzle. Among them there were wrong solutions too. The main mistake in them was that some possible combinations of balls on the balance scales were overlooked, and - as the result - all these solutions don't give the correct and full solution for each of the six balls in any given combination.

Actually there are several different ways how to determine the light and the heavy balls in each of the three pairs. We show some of your solutions to illustrate these ways.

Also we got some solutions which imply that our balance scales can produce different angles (indicated with an arrow) or some distinguishable mutual displacements of the pans, and so this can show that some combinations of balls are "more unequal" or "less unequal" when balls on different pans have different weights.

In fact these solutions can't be fully accepted since in our picture we show simple balance scales which can indicate just whether the both pans hold the same weights or different.

Still these solutions are interesting, but unfortunately the same mistake - overlooked combinations - was in all but one of them. See the last solution below.

A solver called our attention that "... according to Gardner in his "Mathematical Circus" this problem was originally purposed by Paul Curry..." Thanks a lot for this important remark!

Solution by Kiruthika K.

1. Take 2 balls of same color (say blue).
2. Take one ball each of the other 2 colors; and put each one along with a Blue ball on the trays of the balance. (say Blue & White against Blue & Red)
3. There are 2 possibilities:
First:
i) If the balance is equal, then of the Red and the White balls, one is heavy and the other is light.
ii) Keeping track of which Blue ball was paired with which color, weigh the White and the Red balls against each other. Find which one is heavy. (let us say, White turns out to be the heavier one)
iii) Then the Blue weighed with the White is light and the Blue weighed with Red is heavy.

Second:
i) One side is heavier.
ii) The Blue ball on the heavier side is the heavy Blue one.
iii) Put both Blue balls on one side against the White and the Red balls on the other.
iii) If the side with the Blue balls is heavier, then the Red and the White balls are the light ones of their respective colours.
iv) If the other side is heavier, then the Red and the White are both heavy balls.
v) If the balance is equal, the ball that was paired with the heavy Blue ball during the first time is heavy and the other is light.

Example:

Red: R
White: W
Blue: B

1st weighing:
B1R1 Vs B2W1
Equal:
2nd weighing:
R1 Vs W1 (say R1 is heavier, which implies W2 is also heavy)
Then:
B1 is light and B2 is heavy. We know R1 and W2 are heavy therefore R2 and W1 are light.

Solution by David Low

Nothing like a long shower to clear out the cobwebs...

Place a red and blue marble on one side, and a red and white marble on the other.

Case 1. The two sides are the same:

Since there are not four heavy marbles or four light marbles, each side must have one light and one heavy marble. Thus the heavy red marble has a light marble with it, and the light red marble has a heavy marble with it.

For the second massing, just compare the two red marbles. The heavy and light red marbles are directly discovered. Moreover, the marble with the heavy red marble in the first massing is now known to be light, and the marble with the light red marble in the first massing is now known to be heavy. The untouched blue and white marbles will respectively be the opposite of their known same-colour partners.

Case 2. One side is heavier.

The heavier side cannot have the light red marble, since such a situation would give at most one heavy marble on the heavier side, and at least one heavy (red) marble on the lighter side, which is impossible. So the heavier side must have the heavy red marble, and the lighter side has the light red marble. The red marbles are discovered.

For the second massing, place the two red marbles on one side, and the one blue and one white marble from the first massing on the other side. If the blue/white side is heavier than the two reds, both marbles are heavy. If the blue/white side is lighter, both are light. If the balance is the same, then one marble is light and the other heavy. The marble that was with the heavy red in the first massing is heavy and the marble with the light red in the first massing is light, since the reverse would have resulted in a tie in the first massing.

As before, the untouched blue and white marbles will respectively be the opposite of their known same-colour partners.

Solution by Gregory Clayborne

This is a great puzzle. The hardest part is giving the solution in simple steps, but here goes...

Let's first label our balls... R1,R2,W1,W2,B1,B2 and then start weighing...

Weighing 1: Lets weigh R1,W1 vs R2,B1
There are two possible results:
The scales will balance or they won't.

If the scales balance: R1,W1 = R2,B1
then we know that there is a Heavy and a Light on each side. We just don't know who's who. We know this because the Reds can't both be Heavy nor can they both be Light. So we have the following possibilities for R1,W1 = R2,B1:

R1(H),W1(L) = R2(L),B1(H) or
R1(L),W1(H) = R2(H),B1(L)

Notice that this makes W1 the opposite of R1 and B1 the opposite of R2 so.....

Reds opposite Whites R1 = W2, R2 = W1
Reds equal Blues R1 = B1, R2 = B2

For the second weighing we just weigh the Reds against each other and the above equations will finish the results.

If the scales don't balance then it gets a little harder.

We know that R1 can't equal R2 and since the scales didn't balance then we know that
which ever side was Heavy has the Heavy Red ball. Don't believe me do you. Alright.

Let's say that the weighing looked like this:
R1,W1 > R2,B1 thus R1,W1 Heavier than R2,B1. If R1 was actually Light (thus making R2 Heavy) then the only values W1 and B1 could have would be Heavy and Light respectively. BUT that would have given us R1(L),W1(H) > R2(H),B1(L). WHICH WOULD HAVE BALANCED (see L,H = H,L) SO R1 has to be Heavy if it's scale went down.

That being proven, let's stick with the assumption that R1(H) and R2(L) just to make the
logic easier to follow...

The first weighing then gives us the results...
W1 and B1 are the same (both Heavy or both Light) OR
W1 opposite B1 with W1 being the same as R1 and B1 being the same as R2.

This leads us to the final weighing...
We just switch R2 and W1 and wind up weighing R1,R2 vs. W1,B1.

If the scales balance then R1(H) R2(L) and W1(H) B1(L)
If the scales don't balance than R1(H) R2(L) and W1 = B1.

But wait, that doesn't tell us if W1 and B1 are Heavy or Light?
Just answer the question. Did the W1,B1 side of the scale go up or down. There's your answer. The final scale would look like one of the following:

R1(H),R2(L) > W1(L),B1(L) OR
R1(H),R2(L) < W1(H),B1(H)

Ta daaaa.
Like I said. This is a great puzzle. Hope you could get through the logic. I really need to draw it out.

Solution by Shashi

I consider the pair as R1,R2; B1,B2 ; W1,W2

First weighing

R1+W1 against R2+B1
case 1: if it balances then
second weighing,
Weigh R1 and R2 and find out which is heavier, this also tells which of (W1,B1) is heavier.

case 2 if it does not balance
second weighing
weigh R1+R2 against W1+B1
if R1+R2 side goes down then both W1 and B1 are lighter balls of their pairs
if W1+B1 side goes down then both W1 and B1 are heavier balls of their pairs
if it balances
then the ball which was with lighter red ball in the first weighing is lighter i.e., if R1 was lighter then W1 is also lighter

Solution by Marcus Dunstan

6 Balls
White = W1, W2
Red=R1,R2
Blue=B1,B2

First weigh: W1+R1 v W2+B1

If balanced then R1 & B1 must be one HEAVY (H) and one LIGHT (L) (because we know W1 and W2 are one HEAVY and one LIGHT)
If balanced, the second weigh is to swap R1 and B1. ie second weigh is W1+B1 v W2+R1
The side of scales that goes down has 2 HEAVY balls, side of scales that goes up has 2 LIGHT balls.

>From this situation weight of all 6 balls is now known.

If first weigh W1+R1 v W2+B1 is not balanced then you know that the side that goes DOWN must contain the HEAVY white ball eg. W1+R1 v W2+B1
Possible combinations: H+H v L+H, H+H v L+L, H+L v L+L then W1 is HEAVY
or
Possible combinations: L+H v H+H, L+L v H+H, L+L v H+L then W2 is HEAVY
Cannot be combinations: H+L v L+H or L+H v H+L as scales would balance

So at this stage you know the weight of the two white balls only
*** HOWEVER you must also note the colour of the ball on the same side as the LIGHT white ball as this will have bearing depending on the results of the second weigh.

The second weigh would then be W1+W2 v R1+B1
If W1+W2 side goes down then combination must be H+L v L+L or L+H v L+L ie
both R1 and B1are LIGHT
If W1+W2 side goes up then combination must be H+L v H+H or L+H v H+H ie
both R1 and B1 are HEAVY
If W1+W2 balances with R1+B1 then you must have one HEAVY and one LIGHT
ball on the R1+B1 side
The only way this can occur (bearing in mind the results of the first weigh) is if the coloured ball noted above *** was LIGHT

>From this situation weight of all 6 balls is now known.

Solution by Jensen Lai

Label the balls R1, R2, B1, B2, W1 and W2.
Place R1 and B1 on one side of the scale and B2 and W2 on the other side of the scale. There are two possible outcomes. They are equal or they are unequal.

Outcome 1: The two sides are equal.
If two balls on the same side were the same weight, then there would be 4 balls of the same weight. However there are only 3 heavy balls and 3 light ones. Therefore, two balls on the same side, are of different weights.

In the second weighing, weigh R1 and R2. From this weighing it can be determined which red ball is heavy and which is light. Whichever R1 is, B1 is the opposite since they were on the same side in the first weighing. B2 is the opposite of B1. W2 is the
opposite of B2 since they were on the same side of the first weighing and W1 is the opposite of W2. So, if the two sides are equal in the first weighing, then R2, B1 and W2 are of the same weight, and R1, B2 and W1 are of the same weight. The second weighing determines which 3 are heavier and which 3 are lighter.

Outcome 2: The two sides are unequal.
There are four balls on the scales. Two are blue so one of them is lighter and the other one is heavier. Whichever one was on the heavier side must be the heavy blue ball. (The lighter blue ball could not have been on the heavier side because a light blue ball and a heavy other ball is not heavier than a heavy blue ball and a light other ball). The two remaining balls (R1 and W2) are either the same weight or they are different.

In the second weighing, weight R1 against W1. If R1 and W2 are the same weight, R1 and W1 must be different. If R1 and W2 are different weights, R1 and W1 will be equal. So the second weighing can be used to determine whether R1 and W2 are the same or different.

If the second weighing is unequal, R1 and W2 are the same weight and the second weighing will show whether they are heavy or light. R1 is the same as W2 and R2 and W1 are the same. If the second weighing is balanced, then R1 and W2 are different and the heavier one is whichever one was on the heaviest side of the first weighing. The heavier one could not have been with the lighter blue ball or else the first weighing would have been equal and that would be outcome 1 and not outcome 2. R2 is different to R1 and W1 is different to W2 it will be known which balls are heavier and which are lighter.

Solution by Alan Lemm

You have two red (R1, R2), two white (W1, W2) & two blue (B1, B2) balls.
The following are the possibilities for the light and heavy balls (L = light, H = heavy):

B1	R1	W1	B2	R2	W2	CASE #
H	H	H	L	L	L	1
H	H	L	L	L	H	2
H	L	H	L	H	L	3
H	L	L	L	H	H	4
L	H	H	H	L	L	5
L	H	L	H	L	H	6
L	L	H	H	H	L	7
L	L	L	H	H	H	8

Suppose you weigh B1 & R1 (LEFT) against B2 & W1 (RIGHT). The following indicates the result in each case:

CASE #	RESULT
1	LEFT HEAVIER
2	LEFT HEAVIER
3	BALANCED
4	LEFT HEAVIER
5	RIGHT HEAVIER
6	BALANCED
7	RIGHT HEAVIER
8	RIGHT HEAVIER

The second weighing depends upon the result of the first weighing. First, the balanced cases (3 & 6):

You will notice that in each case, R1, B2, and W2 weigh the same as each other, so they are either all the light balls or the heavy balls. Therefore, you weigh R1 against R2.

If R1 is heavier, then you have case 6.
If R1 is lighter, then you have case 3.

Now for the cases where the left side is heavier (1, 2, 4):
In each case, B1 is the heavy ball, and B2 is the light ball. Now you have to contend with the red and white balls. If you weigh R1 against W2, the result will be different for each case.

If R1 is heavier, you have case 1.
If R1 is lighter, then you have case 4.
If the two balls balance, then you have case 2.

Finally, the cases where the right side is heavier (5, 7, 8):

In each case, B1 is the light ball, and B2 is the heavy ball. Again, you have to contend with the red and white balls. If you weigh R1 against W2, the result will again be different for each case.

If R1 is heavier, you have case 5.
If R1 is lighter, then you have case 8.
If the two balls balance, then you have case 7.

You have now determined the weight of each ball in two weighings.

Solution by Tim Sanders

Let's name the balls R1, R2, W1, W2, B1, and B2. Let's assign value 0 to light balls and value 1 to heavy balls. Let's name the scales X and Y.

NOTE: There are 8 possible combinations of weight values for the 6 balls (see table below), and 3 possible outcomes for each weighing - X>Y, X<Y, and X=Y. In choosing the balls to weigh first, the trick is to allocate 3 possible combinations to X>Y, 3 to X<Y, and 2 to X=Y.

Weigh R1-W1 on X and W2-B2 on Y (1st weighing). The following table illustrates the possible combinations for each outcome of the 1st weighing. For instance, if X>Y, then the combinations in rows 1, 2, and 3 are possible.

```
X  X  Y  Y
R1 R2 W1 W2 B1 B2
-----------------
1 0 1 0 1 0 - X>Y
1 0 1 0 0 1 - X>Y
0 1 1 0 1 0 - X>Y
-----------------
0 1 0 1 1 0 - X<Y
0 1 0 1 0 1 - X<Y
1 0 0 1 0 1 - X<Y
-----------------
0 1 1 0 0 1 - X=Y
1 0 0 1 1 0 - X=Y
```

If X>Y, go to (a).
If X<Y, go to (b).
If X=Y, go to ©.

(a) Weigh R1 on X and B1 on Y (2nd weighing).

```
X     Y
R1 R2 W1 W2 B1 B2
-----------------
1 0 1 0 1 0 - X=Y
1 0 1 0 0 1 - X>Y
0 1 1 0 1 0 - X<Y
```

As illustrated above, each possible outcome will indicate the weight value of each ball. For instance, if X=Y, then R1=1, R2=0, W1=1, W2=0, B1=1, and B2=0.

(b) Weigh R1 on X and B1 on Y (2nd weighing).

```
X     Y
R1 R2 W1 W2 B1 B2
-----------------
0 1 0 1 1 0 - X<Y
0 1 0 1 0 1 - X=Y
1 0 0 1 0 1 - X>Y
```

As illustrated above, each possible outcome will indicate the weight value of each ball. For instance, if X<Y, then R1=0, R2=1, W1=0, W2=1, B1=1, and B2=0.

(c) Weigh R1 on X and R2 on Y (2nd weighing).

NOTE: Other ball pairs will work too, but let's use R1-R2 for this solution.

```
X  Y
R1 R2 W1 W2 B1 B2
-----------------
0 1 1 0 0 1 - X<Y
1 0 0 1 1 0 - X>Y
```

As illustrated above, each possible outcome will indicate the weight value of each ball. For instance, if X<Y, then R1=0, R2=1, W1=1, W2=0, B1=0, and B2=1.

Solution by Roland Vyncke

We label the white, red & blue balls as w1, w2 ; r1, r2 & b1, b2 and distinguish the eight possibilities :

L(ight)	H(eavy)	
w1 r1 b1	w2 r2 b2	case 1
w1 r1 b2	w2 r2 b1	case 2
w1 r2 b1	w2 r1 b2	case 3
w1 r2 b2	w2 r1 b1	case 4
w2 r1 b1	w1 r2 b2	case 5
w2 r1 b2	w1 r2 b1	case 6
w2 r2 b1	w1 r1 b2	case 7
w2 r2 b2	w1 r1 b1	case 8

1ST WEIGHING : w1 & b1 against w2 & r1

a) if w1+b1=w2+r1 then

2ND WEIGHING : w1 against w2

if w1 < w2 then case 2
if w1 > w2 then case 7

b) if w1+b1<w2+r1 then
2ND WEIGHING : w1 & w2 against b1 & r1

if w1+w2=b1+r1 then case 3
if w1+w2<b1+r1 then case 4
if w1+w2>b1+r1 then case 1

c) if w1+b1>w2+r1 then
2ND WEIGHING : w1 & w2 against b1 & r1

if w1+w2=b1+r1 then case 6
if w1+w2<b1+r1 then case 8
if w1+w2>b1+r1 then case 5

Remark: b2 & r2 remaining untouched during the whole experiment, a NEW puzzle may be formulated using only 4 balls!!!

Solution by Rob Farley

Ok, suppose we number the balls R1,R2,W1,W2,B1,B2. What we need to do is identify which of the balls are 'L', and which are 'H'.

We have eight possibilities:

1) R1=L,R2=H,B1=L,B2=H,W1=L,W2=H
2) R1=L,R2=H,B1=L,B2=H,W1=H,W2=L
3) R1=L,R2=H,B1=H,B2=L,W1=L,W2=H
4) R1=L,R2=H,B1=H,B2=L,W1=H,W2=L
5) R1=H,R2=L,B1=L,B2=H,W1=L,W2=H
6) R1=H,R2=L,B1=L,B2=H,W1=H,W2=L
7) R1=H,R2=L,B1=H,B2=L,W1=L,W2=H
8) R1=H,R2=L,B1=H,B2=L,W1=H,W2=L

Now... let's do the first weigh.

R1 & B1 against W1 & R2.

If it is even, then we know that we have either case 3 or case 6, and we can distinguish them by weighing R1 against R2 (our second weigh!)

Now, suppose that it went to the right. Then we have either case 1, 2 or 4, and we can distinguish them by weighing B2 and W2.

Similarly, if it went to the right, we have either 5, 7 or 8, which we can also distinguish by weighing B2 and W2.

Solution by William M. Shubert

First, let's call the 6 balls w1, w2, r1, r2, b1, and b2. There are 8 possible combinations:

#	Heavy Balls	Light Balls
1	w1,r1,b1	w2,r2,b2
2	w1,r1,b2	w2,r2,b1
3	w1,r2,b1	w2,r1,b2
4	w1,r2,b2	w2,r1,b1
5	w2,r1,b1	w1,r2,b2
6	w2,r1,b2	w1,r2,b1
7	w2,r2,b1	w1,r1,b2
8	w2,r2,b2	w1,r1,b1

So the goal is to find out which of these combinations is the right one. We have two weighings; each has 3 possible results (left heavier, right heavier, and equal).

Start by weighing w1+r1 (left) vs. w2+b1 (right). Three possible outcomes:

The first weighing leaves the left heavier. The combinations from our list of 8 that leave the left heavier are 1,2, and 4.

Our second weighing is r1 (left) vs. b2 (right). Three outcomes:

Left heavier. Must be combination 1; the heavy balls are w1, r1, and b1.
Equal weight. Must be combination 2; the heavy balls are w1, r1, and b2.
Right heavier. Must be combination 4; the heavy balls are w1, r2, and b2.

The first weighing leaves equal weight. Must be combination 3 or 6.

Our second weighing is r1 (left) vs. r2 (right). Two outcomes:

Left heavier. Must be combination 6; the heavy balls are w2, r1, and b2.
Right heavier. Must be combination 3; the heavy balls are w1, r2, and b1.

The first weighing leaves the right heavier. Must be combination 5, 7, or 8.

Our second weighing is r1 (left) vs. b2 (right). Three outcomes:

Left heavier. Must be combination 5; the heavy balls are w2, r1, and b1.
Equal weight. Must be combination 7; the heavy balls are w2, r2, and b1.
Right heavier. Must be combination 8; the heavy balls are w2, r2, and b2.

There you have it. In all cases, after two weighings I will know exactly which combination of balls I have and which are the heavy ones. There might be an easier or more elegant solution, but this one does work!

Do I win? :-)

Solution by Du'c Hoang

1. weigh R1 & W1 vs R2 & B1.

if balanced

2a. weigh R1 vs R2
if left side is heavier:
R1, B1 & W2 are the heavy balls
if right side is heavier:
R2, B2 & W1 are the heavy balls

if left side is heavier

2b. weigh W2 vs B1
if left side is heavier:
R1, W2 & B2 are the heavy balls
if right side is heavier:
R1, B1 & W1 are the heavy balls
if balanced:
R1, B2 & W1 are the heavy balls (if B1 & W2 are heavy, 1 would be balanced)

if right side is heavier
2c weigh W1 vs B2
if left side is heavier:
R2, W1 & B1 are the heavy balls
if right side is heavier:
R2, B2 & W2 are the heavy balls
if balanced:
R2, B1 & W2 are the heavy balls (if W1 & B2 are heavy, 1 would be balanced)

Solution by Jon Black

Let's call the balls R1, R2, W1, W2, B1, B2.

Put R1 and W1 on one side of the scale and put R2 and B1 on the other side.

If they are equal, then R1 + W1 = R2 + B1 and hence either (R1 > R2, W1< B1, W1 < W2, and B2 < B1) or (R1 < R2,W1 > B1,W1 > W2, and B2 > B1).

Next remove W1 & B1. If R1>R2 then W1 < W2 and B2 < B1. If R1 < R2 then W1 > W2 and B2 > B1.

If R1 + W1 > R2 + B1 then R1 > R2 and hence either (W1 > W2 , B2 < B1) or (W1 > W2, B2 > B1) or (W1 < W2, B2 > B1)

Next put R1 and B1 on one side of the scale and B2 and W2 on the other side. Since R1 > R2, if:

R1 + B1 > B2 + W2, then (B2 < B1, W1 >W2);
R1 + B1 = B2 + W2, then (B2 >B1, W1 >W2);
R1 + B1 < B2 + W2, then (B2 > B1, W1< W2).

If R1 + W1 < R2 + B1 then R1 < R2 and hence either (W1 < W2 , B2 > B1) or (W1 < W2, B2 < B1) or (W1 > W2, B2 < B1)

Next put R1 and B1 on one side of the scale and B2 and W2 on the other side. Since R1 < R2, if:

R1 + B1 < B2 + W2, then (B2 > B1, W1 < W2);
R1 + B1 = B2 + W2, then (B2 < B1, W1 < W2);
R1 + B1 > B2 + W2, then (B2 < B1, W1 > W2).

Solution by Geoffrey Mayne

We'll call the two sides of the balance left and right.

First, have the first white marble and and first blue marble on the left, and the second white marble and the first red marble on the right.

Second, put the second white marble and the first blue marble and the second blue marble and the first red marble on the right.

Based on these weighings, there will be a unique solution of heavier and lighter balls for each possibility.

Solution by James Higgs

"There are three pairs of balls - red, white, and blue. In each pair one ball is a little bit heavier than another one."

So we have r (light) & r' (heavy), w & w' and b & b'. And r' = r + x.

"All the heavy balls weigh the same, and all the light balls weigh the same."

So we have r' = w' = b' and r = w = b.

Weighing 1: Take the two white balls and place one on each side of the balance. This establishes w & w'.

Weighing 2: Put the heavy white ball, w', on the left side of the balance with one of the red balls. On the right side of the balance place the other red ball and one of the blue balls. This produces one of four possible, unique displacements of the balance. If we represent the displacement, D, as being the mass of the left side of the balance minus the right side (D = L - R) then we have the following combinations:

```
L      R      D
=================
w'+ r'  b + r  2x (balance displaced twice w' vs w)
w'+ r'  b'+ r   x (same as w' vs w)
w'+ r   b + r'  0 (balance balanced)
w'+ r   b'+ r' -x (right side heavier)
```