

Treasure of Classic  
and Modern Puzzles

## Puzzles with Numbers

$$a + b = c$$

$$a \times b = c$$

$$2 + 2 = 4$$

$$\square + \square = \square$$

$$2 \times 2 = 4$$

$$\square \times \square = \square$$

### The abc Arithmetics

by Sam Loyd

As it is known 2 multiplied by 2 produces the same result as 2 added to 2. It is 4 in both cases - just as shown in the lower left corner of the illustration.

Can you think of another pair of numbers (**a** and **b**) which when multiplied together give the same result when they are added together. In other words it should be such **a** and **b** numbers that the two equations shown in the illustration are true, i.e. **a + b = c** and **a x b = c**.

*Note*, that the number **a** may be different from the number **b** in the pair. And, of course, you are allowed to choose from the decimal numbers as well.

Actually there are many such pairs and some formula can be derived for this. Can you think of it too?

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## Puzzles with Numbers

$$3 + 1\frac{1}{2} = 4\frac{1}{2}$$
$$3 \times 1\frac{1}{2} = 4\frac{1}{2}$$

### The abc Arithmetics (solution)

The formula which helps to find such pairs is derived from the simple equality that  $a + b = a \times b$ . From this we have that  $b$  always equals  $a/(a-1)$ . The formula shows there is an infinity number of such pairs existing.

Thus if  $a$  is 3 then according to that formula above  $b$  is 1.5. When they are multiplied the result is the same as when they are added together. It is 4.5 as shown in the illustration.