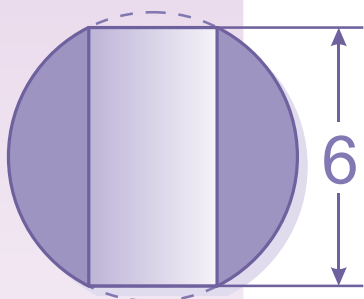


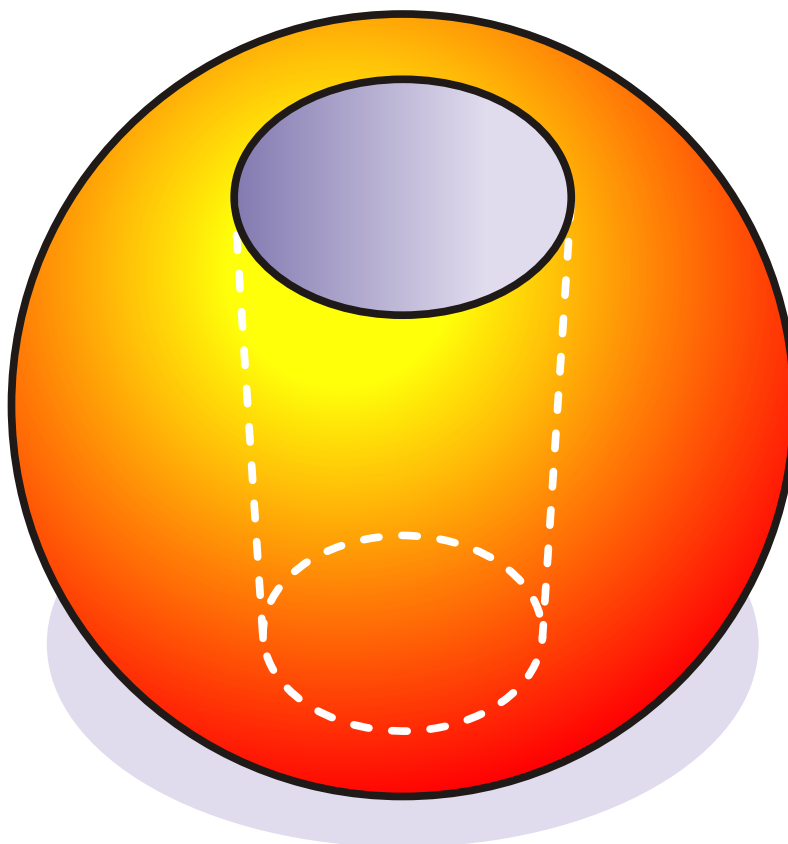
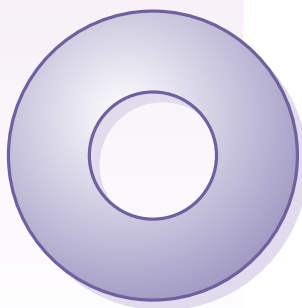
Treasure of Classic  
and Modern Puzzles

## Geometrical Puzzles



Cross section

Top view



### Hole in the Sphere

by Samuel I. Jones\*

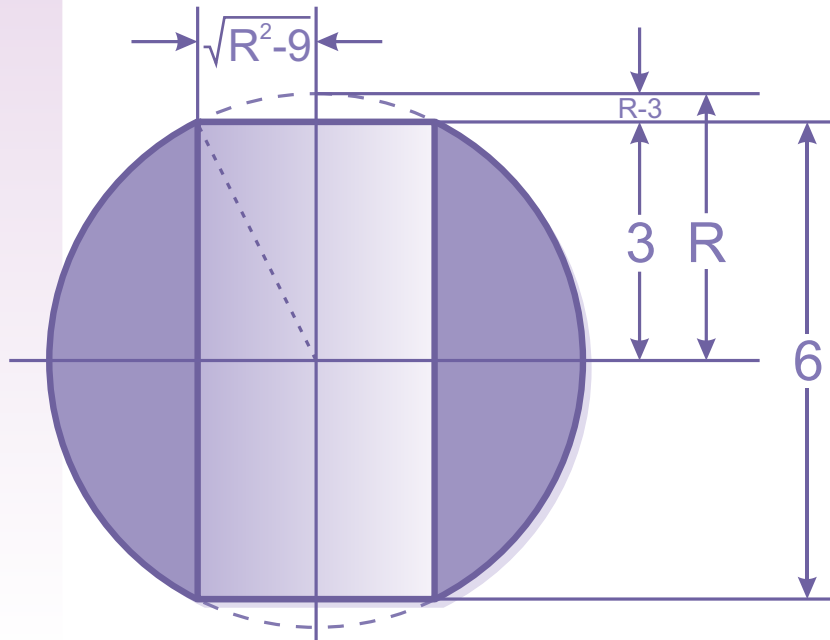
A cylindrical hole six inches long has been drilled straight through the center of a solid sphere - just as shown in the illustration.

What is the volume remaining in the sphere?

-----  
\*Martin Gardner has found the earliest reference for this problem in Samuel I. Jones's *Mathematical Nuts*, self-published, Nashville, 1932.

Treasure of Classic  
and Modern Puzzles

## Geometrical Puzzles



### Hole in the Sphere (solution)

If you want to avoid the calculations in Solution 1, simply take a look directly at Solution 2 below it.

**Solution 1.** Let  $R$  be the radius of the sphere. As the illustration indicates, the radius of the cylindrical hole will then be the square root of  $R^2 - 9$ , and the altitude of the spherical caps at each end of the cylinder will be  $R - 3$ . To determine the residue after the cylinder and caps have been removed, we add the volume of the cylinder,  $6\pi(R^2 - 9)$ , to twice the volume of the spherical cap, and subtract the total from the volume of the sphere,  $4\pi R^3/3$ . The volume of the cap is obtained by the following formula, in which  $A$  stands for its altitude and  $r$  for its radius:  $\pi A(3r^2 + A^2)/6$ .

When this computation is made, all terms obligingly cancel out except  $36\pi$  - the volume of the residue in cubic inches. In other words, the residue is constant regardless of the hole's diameter or the size of the sphere!

**Solution 2.** John W. Campbell, Jr., editor of *Astounding Science Fiction*, was one of several readers who solved the sphere problem quickly by reasoning adroitly as follows: The problem would not be given unless it has a unique solution. If it has a unique solution, the volume must be a constant which would hold even when the hole is reduced to zero radius. Therefore the residue must equal the volume of a sphere with a diameter of six inches, namely  $36\pi$ .

December 7, 2003

www.puzzles.com

Copyright © 2003 ThinkFun Inc. All Rights Reserved. webmaster@ThinkFun.com  
Permission is granted for personal use only. This puzzle may not be duplicated for personal profit.